## Change From:

## 5 Dimension of Equidistribution

Table 3 lists the dimension defects $d(v)$ of SFMT19937 (as a 32-bit integer generator) and of MT19937, for $v=1,2, \ldots, 32$. SFMT has smaller values of the defect $d(v)$ at 26 values of $v$. The converse holds for 6 values of $v$, but the difference is small. The total dimension defect $\Delta$ of SFMT19937 as a 32 -bit integer generator is 4188 , which is smaller than the total dimension defect 6750 of MT19937.

| $v$ | MT | SFMT | $v$ | MT | SFMT | $v$ | MT | SFMT | $v$ | MT | SFMT |
| :---: | ---: | ---: | :---: | ---: | ---: | :---: | :---: | ---: | :---: | ---: | ---: |
| $d(1)$ | 0 | 0 | $d(9)$ | 346 | 1 | $d(17)$ | 549 | 543 | $d(25)$ | 174 | 173 |
| $d(2)$ | 0 | $*_{2}$ | $d(10)$ | 124 | 0 | $d(18)$ | 484 | 478 | $d(26)$ | 143 | 142 |
| $d(3)$ | 405 | 1 | $d(11)$ | 564 | 0 | $d(19)$ | 426 | 425 | $d(27)$ | 115 | 114 |
| $d(4)$ | 0 | $*_{2}$ | $d(12)$ | 415 | 117 | $d(20)$ | 373 | 372 | $d(28)$ | 89 | 88 |
| $d(5)$ | 249 | 2 | $d(13)$ | 287 | 285 | $d(21)$ | 326 | 325 | $d(29)$ | 64 | 63 |
| $d(6)$ | 207 | 0 | $d(14)$ | 178 | 176 | $d(22)$ | 283 | 282 | $d(30)$ | 41 | 40 |
| $d(7)$ | 355 | 1 | $d(15)$ | 83 | $* 85$ | $d(23)$ | 243 | 242 | $d(31)$ | 20 | 19 |
| $d(8)$ | 0 | $* 1$ | $d(16)$ | 0 | $* 2$ | $d(24)$ | 207 | 206 | $d(32)$ | 0 | $* 1$ |

Table 3: Dimension defects $d(v)$ of MT19937 and SFMT19937 as a 32-bit integer generator. The mark * means that MT has a smaller defect than SFMT at that accuracy.

We also computed the dimension defects of SFMT19937 as a 64 -bit (128-bit) integer generator, and the total dimension defect $\Delta$ is 14089 (28676, respectively). In some applications, the distribution of LSBs is important. To check them, we inverted the order of the bits (i.e. the $i$-th bit is exchanged with the ( $w-i$ )-th bit) in each integer, and computed the total dimension defect. It is 10328 (21337, 34577 , respectively) as a 32 -bit ( 64 -bit, 128 -bit, respectively) integer generator. Throughout the experiments, $d^{\prime}(v)$ is very small for $v \leq 11$. We consider that these values are satisfactorily small, since they are comparable with MT for which no statistical deviation related to the dimension defect has been reported, as far as we know.

## Change To:

## 5 Dimension of Equidistribution

Table 3 lists the dimension defects $d(v)$ of SFMT19937 (as a 32-bit integer generator) and of MT19937, for $v=1,2, \ldots, 32$. SFMT has smaller values of the defect $d(v)$ at 25 values of $v$. The converse holds for $\mathbf{7}$ values of $v$, but the difference is small. The total dimension defect $\Delta$ of SFMT19937 as a 32-bit
integer generator is 4199 , which is smaller than the total dimension defect 6750 of MT19937.

| $v$ | MT | SFMT | $v$ | MT | SFMT | $v$ | MT | SFMT | $v$ | MT | SFMT |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $d(1)$ | 0 | * | $d(9)$ | 346 | 3 | $d(17)$ | 549 | 543 | $d(25)$ | 174 | 173 |
| $d(2)$ | 0 | *3 | $d(10)$ | 124 | 1 | $d(18)$ | 484 | 478 | $d(26)$ | 143 | 142 |
| $d(3)$ | 405 | 1 | $d(11)$ | 564 | 0 | $d(19)$ | 426 | 425 | $d(27)$ | 115 | 114 |
| $d(4)$ | 0 | *2 | $d(12)$ | 415 | 117 | $d(20)$ | 373 | 372 | $d(28)$ | 89 | 88 |
| $d(5)$ | 249 | 3 | $d(13)$ | 287 | 285 | $d(21)$ | 326 | 325 | $d(29)$ | 64 | 63 |
| $d(6)$ | 207 | 2 | $d(14)$ | 178 | 176 | $d(22)$ | 283 | 282 | $d(30)$ | 41 | 40 |
| $d(7)$ | 355 | 1 | $d(15)$ | 83 | *85 | $d(23)$ | 243 | 242 | $d(31)$ | 20 | 19 |
| $d(8)$ | 0 | *2 | $d(16)$ | 0 | *2 | $d(24)$ | 207 | 206 | $d(32)$ | 0 | *3 |

Table 3: Dimension defects $d(v)$ of MT19937 and SFMT19937 as a 32-bit integer generator. The mark * means that MT has a smaller defect than SFMT at that accuracy.

We also computed the dimension defects of SFMT19937 as a 64 -bit (128-bit) integer generator, and the total dimension defect $\Delta$ is 14095 (28676, respectively). In some applications, the distribution of LSBs is important. To check them, we inverted the order of the bits (i.e. the $i$-th bit is exchanged with the ( $w-i$ )-th bit) in each integer, and computed the total dimension defect. It is 10336 (21341, 34577, respectively) as a 32 -bit (64-bit, 128-bit, respectively) integer generator. Throughout the experiments, $d^{\prime}(v)$ is very small for $v \leq 11$. We consider that these values are satisfactorily small, since they are comparable with MT for which no statistical deviation related to the dimension defect has been reported, as far as we know.

