Towards the number of tritangents to a smooth sextic

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Consider a smooth degree $2n$ model $X \to \mathbb{P}^{n+1}$ of a $K3$-surface X and denote by $\operatorname{Fn} X$ the *Fano graph* of X, i.e., the adjacency graph of the set of straight lines contained in X. (If the model is hyperelliptic, $X \to Q \hookrightarrow \mathbb{P}^{n+1}$, its smoothness is understood as that of the ramification locus $C \subset Q$, and *lines* in X are smooth rational curves mapped isomorphically to lines in Q.) At present, we know sharp upper bounds on the number $|\operatorname{Fn} X|$ of lines and a detailed description of all close to maximal Fano graphs for all models, both birational and hyperelliptic, of all degrees $2n \geqslant 4$. (Certainly, the best known is the classical case $2n = 4$ of spatial quartics, going as far back as to F. Schur, 1882, and almost settled by B. Segre in 1943.) Thus, the only case still remaining open is that of degree 2 models $X \to \mathbb{P}^2$: each such model is hyperelliptic, the ramification locus is a smooth sextic curve $C \subset \mathbb{P}^2$, and the lines in X are in a two-to-one correspondence with the tritangents to C, i.e., lines tangent to C at three points (which may collide). Here, the Fano graph is so complicated that it is not even clear in what terms one can start describing the *stars* of its vertices, leave alone the whole graph.

In this talk, after a brief introduction to the history of the subject and a few known results, I will motivate the conjecture that $|\operatorname{Fn} X| \leq 144$ for any smooth model of degree 2 (i.e., a smooth sextic curve $C \subset \mathbb{P}^2$ has at most 72 tritangents) and outline a program that would hopefully lead us to a proof of this conjectural sharp bound.

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