Various problems in microlocal analysis and asymptotic analysis

November 6–10, 2023

Room 111, RIMS, Kyoto University

Program

November 6 (Monday)

13:15 – 14:00 **Yutaka Matsui** (Kindai University)
A nesting property of multicomplex hyperfunctions

14:15 – 15:00 **Shunya Adachi** (Chiba University)
On the Laplace transform for KZ type equations

15:15 – 16:00 **Saiei-Jaeyeong Matsubara-Heo** (Kumamoto University)
Non-rigid and globally analyzable hypergeometric system

16:15 – 16:45 **Ken Shibusawa** (Chiba University)
On a power series expansion of the generalized confluent hypergeometric function of Kummer type
(Joint work with Kazuki Hiroe and Yasunori Okada)

November 7 (Tuesday)

10:00 – 10:45 **Tomohiro Asano** (Kyoto University)
Sheaf theoretic characterization of "heaviness"

11:00 – 11:45 **Tasuki Kinjo** (Kyoto University)
Microlocal geometry and derived algebraic geometry

13:15 – 14:00 **Daichi Komori** (Kindai University)
Toward the symbol theory of multi-microlocal operators
(joint work with Naofumi Honda)

14:15 – 15:00 **Luca Prelli** (University of Padova)
Subanalytic sheaves and cosheaves

15:15 – 15:45 **Tatsuki Nishida** (Hokkaido University)
Soft sheaves on subanalytic sites and tempered distributions

16:00 – 16:30 **Ryosuke Sakamoto** (Hokkaido University)
The stalk formula for the multi-microlocal Hom functor and the multi-microlocal Sato triangle

November 8 (Wednesday)

10:00 – 10:45 **Yasunori Okada** (Chiba University)
Slice regular ordinary differential equations and asymptotic analysis
(joint work with Sumito Toyoshima)

11:00 – 11:45 **Shofu Uchida** (Kindai University)
Toward the exact WKB analysis for Appell's hypergeometric system E_2

13:15 – 14:00 **Hidetoshi Tahara** (Sophia University) (Online)
Asymptotic existence theorem for formal power series solutions of singularly perturbed linear q*-difference equations*

14:15 – 15:00 **Reinhard Schäfke** (University of Strasbourg)
Remarks on formal solutions of the singularly perturbed 1-dimensional Schrödinger equation

15:15 – 16:00 **Takashi Aoki** (Kindai University)
Exact WKB analysis of the integral with an elliptic-function phase
(Collaborator: Shofu Uchida (Kindai University))

18:00 – **Banquet**

November 9 (Thursday)

10:00 – 10:45 **Yousuke Ohyama** (Tokushima University)
q*-connection spaces of* q*-Painlevé equations*

11:00 – 11:45 **Frédéric Fauvet** (University of Strasbourg)
Resurgence and the bridge equation with several critical times

13:15 – 14:00 **Akishi Ikeda** (Josai University)
Bimoulds, scrambling operators and singularly perturbed systems

14:15 – 15:00 **David Sauzin** (IMCCE)
Resurgent Poisson structures and deformations for algebras of resurgent series

15:15 – 16:00 **Yoshitsugu Takei** (Doshisha University)
On the exact WKB analysis for difference equations

16:15 – 16:45 **Haru Negami** (Chiba University)
Constrution of representations of braid group and unitarity
(joint work with K. Hiroe)

November 10 (Friday)

10:00 – 10:45 **Tatsuki Kuwagaki** (Kyoto University)
Fukaya category and exact WKB analysis

11:00 – 11:45 **Kento Osuga** (University of Tokyo)
Refined correspondence between Hurwitz numbers and topological recursion

13:15 – 14:00 **Yota Shamoto** (Waseda University)
Stokes structure of difference modules

14:15 – 15:00 **Yuichi Ike** (Kyushu University)
Sheaf quantization for the completion of the space of Lagrangians

15:15 – 15:45 **Yumiko Takei** (Ibaraki College)
Systems of difference-differential equations and integral representations of solutions for confluent family of hypergeometric functions

16:00 – 16:30 **Takahiro Shigaki** (Kwansei Gakuin University)
Exact WKB analysis of nonlinear eigenvalue problems for a certain first order equation

Organizers:
Sampei Hirose (Shibaura Institute of Technology)
Kohei Iwaki (University of Tokyo)
Shingo Kamimoto (Hiroshima University)
Shinji Sasaki (Shibaura Institute of Technology)

Abstract

November 6 (Monday)

Yutaka Matsui (Kindai University)

"A nesting property of multicomplex hyperfunctions"

Multicomplex numbers and the analysis of functions defined on them have been studied recently by D. C. Struppa et. al. In particular, a notion of multicomplex hyperfunctions is introduced by Vajiac-Vajiac [1]. In this talk, we study a nesting property of multicomplex hyperfunctions based on idempotent representations of them.

References

[1] A. Vajiac and M. Vajiac, Multicomplex hyperfunctions, Complex Var. Elliptic Equ. 57 (2012), 751-762.

Shunya Adachi (Chiba University)

"On the Laplace transform for KZ type equations"

KZ (Knizhnik-Zamolodchikov) type equation is a class of regular holonomic systems. Recently, the study of KZ type equations via the middle convolution (Euler transform) has been actively developed by Haraoka, Oshima, and Matsubara, etc. Their work brings a new development in the global analysis of regular holonomic systems from the view point of integral transformations.

In this talk, I will explain my ongoing work on the Laplace transform for KZ type equations. In particular, how the Laplace transform changes the equations and the irreducibility of the transformed equations will be discussed. While the middle convolution sends a KZ type equation to another KZ type equation, our transform sends a KZ type equation to an irregular holonomic system. It is expected that our result will be applicable to the global analysis of irregular holonomic systems, such as the confluent hypergeometric system in several variables.

Saiei-Jaeyeong Matsubara-Heo (Kumamoto University)

"Non-rigid and globally analyzable hypergeometric system"

Middle convolution is a class of invertible transformation of a Fuchsian ODE introduced by Nicolas Katz around 90's. As it is an integral transformation with a simple kernel (Kummer local system), one can analyze how the monodromy representation changes via this transformation. This reveals a nature of classical hypergeometric system and lead to discovery of several new classes of globally analyzable Fuchsian ODE. In 2012, Haraoka introduced middle convolution for KZ(-type) system, which initiated a new study on KZ system. In this talk, we report on a class $E^{p,q,r}_{p,q,r}$ of hypergeometric system in several variables. This class of system, viewed as an ODE in a particular direction, is not rigid in general, yet globally analyzable in the sense that one can compute its global monodromy. The system can also be regarded as a particular case of a larger family $E^{p,q,r}_{p',q',r'}$, which is also a special class of Cohen-Macaulay GKZ system. This is based on an on-going joint work with Toshio Oshima.

Ken Shibusawa (Chiba University)

"On a power series expansion of the generalized confluent hypergeometric function of Kummer type"

(Joint work with Kazuki Hiroe and Yasunori Okada)

Let $r \in \mathbb{Z}_{\geq 2}$ be an integer equal to or greater than 2, $N \in \mathbb{Z}_{\geq r}$ an integer equal to or greater than r, and $\lambda = (n_1, n_2, \cdots, n_\ell)$, $n_1 + n_2 + \cdots + n_\ell = N$ be a partition of N. For r and λ, Kimura, Haraoka, and Takano[1] gave the generalized confluent hypergeometric function $\Phi_{r,\lambda}(z;\alpha;\gamma)$ as a generalization from classical hypergeometric and confluent hypergeometric functions. The Kummer type

$$\begin{aligned} \Phi_{(2,1,1)}(z;\alpha;\gamma) := \Phi_{2,(2,1,1)}(z;\alpha;\gamma) &= \int_\gamma \chi_{(2,1,1)}(tz;\alpha)(t_0 dt_1 - t_1 dt_0), \\ \chi_{(2,1,1)}(tz;\alpha) &= (tz_1)^{\alpha_1} e^{\alpha_2 \frac{tz_2}{tz_1}} (tz_3)^{\alpha_3} (tz_4)^{\alpha_4} \\ &\qquad (tz \in \tilde{H}_{(2,1,1)}) \end{aligned} \tag{1}$$

was treated in a more detailed way in Kimura[3], where the variable $z \in Z_{(2,1,1)}$ is in the set defined by

$$Z_{(2,1,1)} = \left\{ z = (z_1\ z_2\ z_3\ z_4) = \begin{pmatrix} z_{0,1} & z_{0,2} & z_{0,3} & z_{0,4} \\ z_{1,1} & z_{1,2} & z_{1,3} & z_{1,4} \end{pmatrix} \in \mathrm{Mat}(2,4) : \det(z_i, z_j) \neq 0, (i,j) = (1,2),(1,3),(1,4),(3,4) \right\},$$

the parameter α is a row vector written as

$$\alpha = (\alpha_1, \alpha_2, \alpha_3, \alpha_4), \quad \alpha_1 + \alpha_3 + \alpha_4 = -2,$$

the integral path γ is a curve which is a function of $t = (t_0, t_1) \in \mathbb{P}^1$ with the two end points $[t_0 : t_1] = [-z_{1,i} : z_{0,i}], [-z_{1,j} : z_{0,j}]$, $(i, j = 1, 3, 4)$: γ connects two zeros of the integrand, and the set $\tilde{H}_{(2,1,1)}$ is a universal cover of the group

$$H_{(2,1,1)} = \left\{ \left(\begin{array}{cc|c|c} h_1 & h_2 & & \\ & h_1 & & \\ \hline & & h_3 & \\ \hline & & & h_4 \end{array} \right) \in \mathrm{GL}(4) : h_1, h_3, h_4 \neq 0 \right\}.$$

The function $\Phi_{(2,1,1)}(z; \alpha; \gamma)$ is a solution of the confluent hypergeometric system of Kummer type $M_{(2,1,1)}(\alpha)$ by Kimura, Haraoka, and Takano[2]. Here we use the notation $M_{(2,1,1)}(\alpha)$ without defining since we see the function $\Phi_{(2,1,1)}(z; \alpha; \gamma)$ as a main theme of the talk.

In the talk, we give power series expansions of the functions $\Phi_{(2,1,1)}(z; \alpha; \gamma)$ defined in an integral as (1) and $\tilde{\Phi}_{(2,1,1)}(z; \alpha, \beta; \gamma)$ defined by

$$\tilde{\Phi}_{(2,1,1)}(z; \alpha, \beta; \gamma) = \int_\gamma \chi_{(2,1,1)}(tz; \alpha) \log \chi_{(2,1,1)}(tz; \beta)(t_0 dt_1 - t_1 dt_0) \quad (tz \in \tilde{H}_{(2,1,1)}), \tag{2}$$

where $\beta = (\beta_1, \beta_2, \beta_3, \beta_4)$, $\beta_1 + \beta_3 + \beta_4 = 0$ is one of the parameters of $\tilde{\Phi}_{(2,1,1)}(z; \alpha, \beta; \gamma)$. Those two are the entries of a vector-valued function

$$\hat{\Phi}^{[2]}_{(2,1,1)}(z; \alpha, \beta; \gamma) = \begin{pmatrix} \tilde{\Phi}_{(2,1,1)}(z; \alpha, \beta; \gamma) \\ \Phi_{(2,1,1)}(z; \alpha; \gamma) \end{pmatrix}. \tag{3}$$

The function $\hat{\Phi}^{[2]}_{(2,1,1)}(z; \alpha, \beta; \gamma)$ is a solution of the system of differential equations $\hat{M}^{[2]}_{(2,1,1)}(\alpha, \beta)$ which can be considered as an extension from $M_{(2,1,1)}(\alpha)$. We made $\hat{M}^{[2]}_{(2,1,1)}(\alpha, \beta)$ to obtain extended contiguity relations from the ones for $M_{(2,1,1)}(\alpha)$ given by Kimura, Haraoka, and Takano[2], but, in the talk, we focus how those expansions can be done. For our expansions, the formulas

$$\begin{array}{rcll} \Phi_{(2,1,1)}(gz; \alpha; (g^{-1})_* \gamma) & = & (\det g)^{-1} \; \Phi_{(2,1,1)}(z; \alpha; \gamma) & (g \in \mathrm{GL}(2)) \\ \Phi_{(2,1,1)}(zh; \alpha; \gamma) & = & \chi_{(2,1,1)}(h; \alpha) \, \Phi_{(2,1,1)}(z; \alpha; \gamma) & (h \in \tilde{H}_{(2,1,1)}) \end{array} \tag{4}$$

by Kimura[3] to transform $\Phi_{(2,1,1)}(z; \alpha; \gamma)$ are used to give the good form

$$\Phi_{(2,1,1)}(z; \alpha; \gamma) = \frac{1}{[4,3]} e^{\alpha_2 \frac{[2,3]}{[1,3]}} \left(\frac{[4,3]}{[4,1]} \right)^{\alpha_3 + 1} \left(\frac{[4,3]}{[1,3]} \right)^{\alpha_4 + 1} \int_a^b e^{\alpha_2 \tilde{x} u} u^{\alpha_3} (1-u)^{\alpha_4} \frac{du}{ds} ds,$$

$$\tilde{x} = \frac{[1,2][3,4]}{[1,3][1,4]} \in \mathbb{C}^\times,$$

where we denote the determinant of the square matrix $(z_i\ z_j)$ of size 2, $\det(z_i, z_j)$ $(1 \le i, j \le 4)$ by $[i,j]$, consider t_0, t_1 as the functions of $s \in [a,b] \subset \mathbb{R}$, and put $u(s) = \frac{t_1(s)}{t_0(s)}$ with the assumption that $u(s)$ has its derivative with respect to s. We call this good form as the normal form of $\Phi_{(2,1,1)}(z;\alpha;\gamma)$, which enables us to obtain easily a power series expansion of $\Phi_{(2,1,1)}(z;\alpha;\gamma)$. The formulas in (4) come from the group actions of GL(2) and $H_{(2,1,1)}$ on the set $Z_{(2,1,1)}$.

References

[1] Kimura, H., Haraoka, Y.,Takano, K.; The generalized hypergeometric functions; *Proc. Japan Acad.* **68**, 290-295; 1992.

[2] Kimura, H., Haraoka, Y.,Takano, K.; On contiguity relations of the confluent hypergeometric systems; *Proc. Japan Acad.* **70**, 47-49; 1994.

[3] Kimura, H.; Introduction to hypergeometric functions (in Japanese); Saiensu-sha; 2007; Tokyo.

November 7 (Tuesday)

Tomohiro Asano (Kyoto University)

"Sheaf theoretic characterization of "heaviness""

Tamarkin provided a method to prove the non-displaceability of subsets of cotangent bundles with respect to Hamiltonian isotopies using microlocal sheaf theory. On the other hand, Entov-Polterovich introduced the concepts of heaviness and superheaviness as more precise concepts leading to non-displaceability.

In this talk, I will provide criteria for (super)heaviness of the subsets of the cotangent bundles via microlocal sheaf theory.

Tasuki Kinjo (Kyoto University)

"Microlocal geometry and derived algebraic geometry"

In this talk, we will discuss an extension of microlocal operations of sheaves on manifolds to derived Artin stacks. As an application, we will provide a microlocal construction of the virtual fundamental class. This talk is based on a joint work with Adeel Khan.

Daichi Komori (Kindai University)

"Toward the symbol theory of multi-microlocal operators"

(joint work with Naofumi Honda)

The formulation of the theory of multi-microlocalization have been studied by Honda and Prelli in recent years. In this talk we introduce multi-microlocal operators and their symbol theory, which are the generalizations of the symbol theory of pseudodifferential operators studied by Aoki. This talk is based on the ongoing joint work with Naofumi Honda.

Luca Prelli (University of Padova)

"Subanalytic sheaves and cosheaves"

Sheaf theory is widely used to study objects which are very useful in various fields of mathematics. What is less known is that there are other objects having a "covariant" nature, which are better described bycosheaves. We will recall the definition of a cosheaf on a topological space and see a link between sheaves and cosheaves. After that we will consider sheaves and cosheaves on the subanalytic site. In this setting new objects can be studied. This is a work in progress with P. Polesello (Padova University).

Tatsuki Nishida (Hokkaido University)

"Soft sheaves on subanalytic sites and tempered distributions"

In this talk, we explain how to represent tempered distributions via Čech-Dolbeault cohomology. Sheaves on subanalytic sites are necessary for tempered distributions. We will define soft sheaves on subanalytic sites and then develop subanalytic Čech-Dolbeault theory.

Ryosuke Sakamoto (Hokkaido University)

"The stalk formula for the multi-microlocal Hom functor and the multi-microlocal Sato triangle"

The concept of multi-microlocalization was introduced to extend the usual microlocal sheaf theory to a more general scope. This talk aims to further extend this theory by exploring advanced topics such as a stalk formula for multi-microlocalized Hom functors and the Sato triangle in the context of multi-microlocal analysis and subanalytic sheaf theory.

November 8 (Wednesday)

Yasunori Okada (Chiba University)

"Slice regular ordinary differential equations and asymptotic analysis"

(joint work with Sumito Toyoshima)

A notion of slice regular functions in one quaternionic variable is introduced by Gentili-Struppa [1, 2]. Polynomials and convergent power series define such functions, and they share some properties similar to those of holomorphic functions. We study slice regular ODEs for slice regular functions, from the viewpoint of asymptotic analysis.

References

[1] G. Gentili and D. C. Struppa. A new approach to Cullen-regular functions of a quaternionic variable. *C. R. Math. Acad. Sci. Paris*, 342(10):741–744, 2006.

[2] G. Gentili and D. C. Struppa. A new theory of regular functions of a quaternionic variable. *Adv. Math.*, 216(1):279–301, 2007.

Shofu Uchida (Kindai University)

"Toward the exact WKB analysis for Appell's hypergeometric system E_2"

We investigate two-dimensional holonomic systems with a large parameter from the viewpoint of the exact WKB analysis. In this talk, we define the Voros coefficient at the origin for Appell's hypergeometric system E_2 and give its explicit form for the system.

Hidetoshi Tahara (Sophia University) (Online)

"Asymptotic existence theorem for formal power series solutions of singularly perturbed linear q-difference equations"

In this talk, we consider singularly perturbed linear q-difference equations with holomorphic coefficients. Under a suitable condition, the equation has a formal power series solution $\hat{u}(t,\varepsilon) = \sum_{n\geq 0} u_n(t)\varepsilon^n$ with respect to the parameter ε. In the case $|q| > 1$, the coefficient $u_n(t)$ is holomorphic in a neighborhood U_n of $t = 0$, but the domain U_n is shrinking (as $n \longrightarrow \infty$) to $\bigcap_{n\geq 0} U_n = \{0\}$. In this situation, it is quite natural to consider the following problem: Can we show the existence of a true solution that admits $\hat{u}(t,x)$ as an asymptotic expansion (as $\varepsilon \longrightarrow 0$). If this is OK, we say that the asymptotic existence theorem is valid.

The purpose of this talk is to show that the asymptotic existence theorem is valid in the case $q > 1$. Since usual Borel summability (or q-summability) method cannot be applied to this case, we apply q-analogues of the argument developed in Tahara (J. Differential Equations, 373 (2023), 283-326).

Reinhard Schäfke (University of Strasbourg)
"Remarks on formal solutions of the singularly perturbed 1-dimensional Schrödinger equation"

Associating actual solutions to formal solutions of the singularly perturbed 1-dimensional Schrödinger equation has been considered for a long time in many works. A few aspects of this topic are mentioned in the lecture.

Takashi Aoki (Kindai University)
"Exact WKB analysis of the integral with an elliptic-function phase"
(Collaborator: Shofu Uchida (Kindai University))

We are interested in the function defined by

$$u = \int \exp(\eta \wp(z)) dz.$$

Here $\wp(z)$ denotes the Weierstrass elliptic function and η a large parameter. The path of integration is chosen suitably. This gives an extension of the Hansen formula for the Bessel function J_0:

$$J_0(z) = \frac{1}{\pi} \int_0^{\pi} \exp(iz \cos\theta) d\theta.$$

As is well known, $\wp$ satisfies the following differential equation expressed in terms of the invariants g_2, g_3:

$$(\wp'(z))^2 = 4\wp(z)^3 - g_2 \wp(z) - g_3.$$

We set $x_1 = -g_2, x_2 = -g_3$ and consider u as a function of x_1, x_2, η. We find a holonomic system of rank 3 which characterizes u and construct WKB solutions to this system. The Borel transform of the WKB solutions can be written explicitly. Using this fact, we can analyze the connection problems for the WKB solutions. If one of the variables, say x_1, is fixed, and η is considered as a parameter, the holonomic system is reduced to a third order example of the higher order simple-pole type equations introduced by Kawai-Koike-Takei.

November 9 (Thursday)

Yousuke Ohyama (Tokushima University)
"q-connection spaces of q-Painlevé equations"

We study connection problems of some linear systems associated with q-Painlevé equations. The connection spaces a q-analogue of the Fricke-Klein cubic space which appears as a character variety of the sixth Painlevé differential equations. At first we review the q-Riemann-Hilbert problem following G. D. Birkhoff.

In the case of q-Painleve VI equation, the connection space is a del Pezzo surface of degree four, so called the Segre surface. We show that geometric aspects of the Segre surface play an important role on the study of analytic behavior of q-Painlevé equations.

Frédéric Fauvet (University of Strasbourg)
"Resurgence and the bridge equation with several critical times"

Divergent series appearing in solutions of complex analytic dynamical systems at general irregular singularities may involve a "mixing of several Gevrey orders of growth", calling for relevant summation mechanisms to eventually describe the Stokes phenomenon. We shall present the resurgent approach for such situations, involving Ecalle's acceleration operators, with a focus on explicit examples for linear differential equations; we will notably explain and illustrate the bridge equation in this context.

Akishi Ikeda (Josai University)
"Bimoulds, scrambling operators and singularly perturbed systems"

The notion of (bi)mould is introduced by J.Écalle in his resurgence theory. In this talk, we summarize and reformulate Écalle's works on bimould and construct certain bimoulds by using Écalle's scrambling operators. We see the role of these bimoulds in singularly perturbed systems and explore resurgence property of them.

David Sauzin (IMCCE)
"Resurgent Poisson structures and deformations for algebras of resurgent series"

I will review the definition of the algebra A of simple Z-resurgent series and its alien derivations Δ_m, as given by Jean Ecalle in 1981. These operators are independent in a strong sense. The freeness of the Lie algebra generated by the Δ_m's under commutators and multiplication by elements of A has an interesting consequence: since we have so many derivations (although we are dealing with a formal series of one variable), one can construct non-trivial Poisson structures on A and, correspondingly, non-commutative deformations of the product of A.

Yoshitsugu Takei (Doshisha University)

"On the exact WKB analysis for difference equations"

In this talk we consider the exact WKB analysis for difference equations. Taking some concrete examples which are related to hypergeometric functions of confluent type, we study their WKB solutions, Stokes curves and connection formulas. We also discuss an alternative appraoch to the exact WKB analysis for difference equations via the Laplace transformation.

Haru Negami (Chiba University)

"Constrution of representations of braid group and unitarity"

(joint work with K. Hiroe)

There are various ways to define braid groups B_n. One is to view it as the fundamental group of the configuration space of unordered n-points on the complex plane, and another is to view it as the mapping class group of a disk with n-points, and so on. The monodromy representation for KZ-type equations is the anti-representation of the pure braid group P_n through the former view. In [1], Haraoka obtained a method to construct a new anti-representation of the P_n from any given anti-representation of the P_n through multiplicative middle convolution of the KZ-type equation.

In this talk, we will apply the Katz-Long-Moody construction, a construction method of representations of braid groups mentioned in [2], to the case of P_n and discuss the correspondence with Haraoka's construction method. We then discuss the further extension of the method and the unitarity of the representations.

References

[1] Y. Haraoka, *Multiplicative middle convolution for KZ equations*, Mathematische Zeitschrift (2020)

[2] K. Hiroe and H. Negami, *Long-Moody construction of braid representations and Katz middle convolution*, https://arxiv.org/pdf/2303.05770.pdf

November 10 (Friday)

Tatsuki Kuwagaki (Kyoto University)
"Fukaya category and exact WKB analysis"

It is expected that some structures that appeared in exact WKB analysis (e.g. Stokes curves/spectral networks, Voros formula, DDP formula) have explanations from the Fukaya category theory. In this talk, I' ll explain partial justifications and speculations of these expectations. This is based on a joint work in progress with Kohei Iwaki and Hiroshi Ohta.

Kento Osuga (University of Tokyo)
Refined correspondence between Hurwitz numbers and topological recursion

It has been proven that one can construct generating functions of Hurwitz numbers from topological recursion on an appropriate spectral curve. In this talk, I will explore this correspondence in a refined setting. On one hand, we have enumerative invariants of maps onto possibly nonorientable surfaces, and on the other hand, we have refined topological recursion on a refined spectral curve. After reviewing both aspects, I will give a sketch of how to prove their correspondence. If time permits, I will mention how far we can extend, and also discuss other applications of refined topological recursion. This talk is partly based on work in progress with Nitin Chidambaram and Maciej Dolega.

Yota Shamoto (Waseda University)
"Stokes structure of difference modules"

P. Deligne introduced the notion of a Stokes-filtered local system, or Stokes structure, to describe the asymptotic behavior of solutions of linear differential equations in one complex variable. We shall present the analogous concept for linear difference equations and formulate the Riemann-Hilbert correspondence under a mild condition on formal structure. If time permits, we will also discuss the further directions.

Yuichi Ike (Kyushu University)
"Sheaf quantization for the completion of the space of Lagrangians"

The space of smooth compact exact Lagrangians of a cotangent bundle carries the spectral metric γ, and we can consider its completion. In this talk, I will explain how we can use sheaf-theoretic methods to explore the completion. I will show that (1) we can associate a sheaf with an element of the completion, and (2) the γ-support and the (reduced) microsupport coincide through the correspondence. This is joint work with Asano, Guillermou, Humilière, and Viterbo.

Yumiko Takei (Ibaraki College)

"Systems of difference-differential equations and integral representations of solutions for confluent family of hypergeometric functions"

Takahiro Shigaki (Kwansei Gakuin University)

"Exact WKB analysis of nonlinear eigenvalue problems for a certain first order equation"

Bender, Fring and Komijani studied a certain first order differential equation as a typical example of nonlinear eigenvalue problems. In their result, an asymptotic behavior of the eigenvalues is obtained.

We try to apply exact WKB analysis to this problem. In our talk, we describe the construction of so-called 0-parameter solution and the Borel summability in some Stokes regions. We also introduce that the eigenfunctions are written by the Borel sum of a 0-parameter solution. If time permits, we discuss our ongoing work.