Integral geometry by sheaves and $\mathcal{D}$-modules

Corrado Marastoni
University of Padova (Italy) and R.I.M.S. of Kyoto University

Hiroshima, January 8 2002

In this talk we shall try to convince the audience, by using a motivational introduction through the Radon and the Penrose transforms, that the approach by sheaves and $\mathcal{D}$-modules is well-suited for the study of integral geometry. Here is a brief sketch of our program.

(1) What are the *Radon* and the *Penrose transforms*? What is behind the fact that *their ranges are characterized essentially by the same system of linear PDEs*?

(2) Brief introduction to *sheaves* (used to describe the geometry and regularity —analytic, $\mathcal{C}^\infty$, ...— of the desired solutions) and *$\mathcal{D}$-modules* (to describe the systems of linear PDEs appearing in the problem).

(3) *Adjunction formulas* : i.e., how do sheaves and $\mathcal{D}$-modules fit together in order to describe the transform?

(4) *Group invariance* : when one deals with compact homogeneous manifolds of some complex semisimple Lie group G (as are, for example, the projective space or, more generally, the Grassmannians for $G = SL(n; \mathbf{C})$), and the problem is G-invariant, things become clearer.

(5) The *concrete case of complex Radon-Penrose transform* : the answer to the second question of (1) is that the Radon and the Penrose transforms are induced —as well as various other classical real or complex problems— by the same overlying complex integral transform.

In performing the above program, we shall use the results obtained in our recent joint works with T. Tanisaki (to appear in "Differential geometry and its Applications") for the $\mathcal{D}$-module side of the problem, and with A. D'Agnolo (Publications R.I.M.S. Kyoto, vol. 36, 2000, p. 337–383) for the sheaf-theoretic part.