Estimation of the noncentrality matrix of a noncentral Wishart distribution with unit scale matrix, employing a matrix loss function

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Consider $S \sim W_m(n, I_m, M'M)$. The habitual unbiased estimator of $M'M$ is $T := S - nI_m$. Under certain conditions $T_\alpha := T + \alpha(\mathrm{tr}S)^{-1}I_m$ is better than T, for suitable α. Leung (1994) showed this using the loss function

$$\lambda[(M'M)^{-1}, R] := \mathrm{tr}\{(M'M)^{-1}R - I_m\}^2.$$

We shall use a *matrix* loss function

$$L[(M'M)^{-1}, R] := \{(M'M)^{-1}R - I_m\}'\{(M'M)^{-1}R - I_m\},$$

and apply Lywner partial ordering of symmetric matrices. An *approximate* domination result will be proved, the error term being of order $o(n^{-1})$. We shall use a matrix version of a Fundamental Identity for the noncentral Wishart distribution. [Leung gave a *scalar* version extending Hass's Fundamental Identity (*scalar* version) for the central Wishart distribution.] A matrix version of Leung's ancillary Lemma 3.1 will then be established. We shall employ an approximation of $\mathcal{E}(\mathrm{tr}S)^{-1}S, \mathcal{E}$ being the expectation operator. A lemma of the matrix Hessian $\bigtriangledown\varphi F$, where $\varphi(F)$ is a scalar (matrix) function of S will be proved. Further a lemma on the *scalar* Hessian $\mathrm{tr}\bigtriangledown F_2AF_1$, where F_1 and F_2 are matrix functions of S and A is a constant matrix, will be given. **References**: Hass, L. R. (1981) Canadian J. Statist, 215-224. Leung, P. L. (1994) J. Multivariate Anal. 48, 107–14.