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Open books on 5-dimensional manifolds

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ABSTRACT. We first give a necessary and sufficient condition for two open books on a rational homology 5-sphere to have equivalent bindings, which generalizes a result in [22] for open books on S^5. Second we give an existence theorem of an open book realizing a given 4-manifold as its page, up to taking connected sum with some copies of $S^2 \times S^2$. Finally, as an application of all these results, we give an algebraic criterion for two diffeomorphisms of a compact simply connected 4-manifold with boundary to be isotopic up to connected sum with some copies of $S^2 \times S^2$, which is similar to a result of Quinn [19] for closed 4-manifolds.

1. Introduction

Let M be a smooth closed manifold and K a codimension two submanifold with trivial normal bundle. We assume that the complement $M \setminus K$ admits a smooth locally trivial fibration ϕ over S^1 which is consistent with the normal bundle of K. Then the triad (M, K, ϕ) is called an *open book* (for details, see Definition 2.1). Furthermore, K is called the *binding*, and the closure in M of each fiber of the fibration is called a *page*, which can be regarded as a Seifert manifold of K.

Open book structures on odd dimensional manifolds have been extensively studied and various important results have been obtained (for example, see [5], [6], [12], [17], [25], [27]). However, almost all the results have concerned open books on manifolds whose dimensions are greater than or equal to 7. It has been observed that, even when the ambient manifold M is diffeomorphic to the sphere, the situation is totally different for dimension 5 (for details, see [20], [21], [22]).

When $M = S^5$, many results have been obtained in [20], [21], [22]. In this paper, we generalize some of the results in [22] to open books on 1-connected 5-dimensional manifolds which are not necessarily spheres. We note that in higher dimensions, a classification of certain open books on highly connected manifolds which are not necessarily spheres has been obtained in [15].

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In §2, we give a necessary and sufficient condition for two open books on a 5-manifold M to have equivalent bindings up to diffeomorphisms of M, or to be topologically equivalent as open books, by using Seifert linking forms. It has been known that such a linking form plays an important role in the study of open books on *spheres* ([5], [6], [20], [21], [22]). When the ambient manifold is not necessarily a sphere, one cannot define such a linking form over the *integers* in general. However, we shall consider 1-connected 5-manifolds M with $H_*(M;\mathbf{Q}) \cong H_*(S^5;\mathbf{Q})$ as the ambient manifold, so that we can define such linking forms over the *rational numbers*.

In §3, we consider smooth equivalence of open books on 5-dimensional manifolds, allowing ourselves to take connected sum with some copies of a certain standard open book on S^5 whose page is diffeomorphic to $(S^2 \times S^2)\sharp(S^2 \times S^2)\setminus \mathrm{Int}\, D^4$. Such an equivalence is called a stable equivalence.

In §4, we give a realization result of a given compact 1-connected 4-manifold with boundary as a page of some open book on standard 5-manifolds. In general, it is difficult to realize a given manifold itself, and we will allow ourselves to take connected sum with some copies of $S^2 \times S^2$. The ambient 5-manifold will be the 5-sphere when the given 4-manifold is spin, and it will be the standard 1-connected nonspin 5-manifold X_{-1} with $H_2(X_{-1};\mathbf{Z}) \cong \mathbf{Z}_2$ (see [2]) when it is not spin.

In §5, as an application of the results obtained in the previous sections, we give an algebraic criterion for two diffeomorphisms of a compact 1-connected 4-manifold which are the identity on the boundary to be stably isotopic, where two such diffeomorphisms h_0 and h_1 of a 4-manifold F are *stably isotopic* if $h_0\sharp k(\mathrm{id})$ and $h_1\sharp k(\mathrm{id})$ are smoothly isotopic relative to boundary as diffeomorphisms of $F\sharp k(S^2 \times S^2)$ for some $k \geq 0$ (see [22, Conjecture 9.6]). Note that such a criterion has already been obtained in [22, Proposition 9.5] when F is spin, by using open books on the 5-sphere. When F is not spin, we follow a similar argument, but we use open books on the rational homology 5-sphere X_{-1}.

Throughout the paper, we work in the smooth category unless otherwise specified. All the homology and cohomology groups are with integer coefficients unless otherwise indicated. We use the symbol "$\cong$" to denote a diffeomorphism between smooth manifolds or an appropriate isomorphism between algebraic objects.

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2. Equivalence of bindings and topological equivalence

Let us first define simple open books on a 5-dimensional manifold M and their rational Seifert forms when M is a rational homology sphere.

Definition 2.1. Let K be a closed 3-manifold smoothly embedded in a closed 5-manifold M with trivial normal bundle. Suppose that there exist a smooth fibration $\phi : M \backslash K \to S^1$ and a trivialization $\tau : K \times D^2 \to N(K)$ of a tubular neighborhood $N(K)$ of K in M such that the following diagram is commutative, where p denotes the obvious projection:

$$\begin{array}{ccc} K \times (D^2 \backslash \{0\}) & \xrightarrow{\ \tau\ } & N(K) \backslash K \\ & {\scriptstyle p} \searrow \quad \swarrow {\scriptstyle \phi} & \\ & S^1. & \end{array}$$

Then we call the triad (M, K, ϕ) an *open book*. Furthermore, we call K the *binding*. Note that the closure in M of each fiber of ϕ is a compact 4-manifold embedded in M whose boundary is identified with K; i.e., it can be regarded as a Seifert manifold of K. Such a 4-manifold is called a *page*.

An open book (M, K, ϕ) is *simple* if the pages of ϕ are 1-connected and K is connected. Note that in this case, the 5-manifold M is simply connected.

In this paper, we always assume that open books are simple. Thus in the following we often omit the word "simple" for simplicity. Furthermore, we always assume that S^1 and M are oriented. Note that then every page and its boundary K have canonical orientations.

Remark 2.2. For every closed simply connected 5-manifold M, there exists a simple open book on M [1].

From now on, we assume that M is a simply connected rational homology 5-sphere; i.e., we assume $H_*(M; \mathbf{Q}) \cong H_*(S^5; \mathbf{Q})$. Note that in this case, $H_2(M)$ is finite and $H_3(M) = 0$. Let us define the bilinear form

$$\Gamma_K : H_2(F) \times H_2(F) \to \mathbf{Q}$$

as follows, where F is a page of a simple open book (M, K, ϕ).

Let c_1 and c_2 be two disjoint 2-cycles in M. Since $H_2(M; \mathbf{Q}) = 0$, there exist a nonzero integer m and a 3-chain $\tilde{c}_1$ in M such that $\partial \tilde{c}_1 = m c_1$. Then we define the *rational linking number* $\mathrm{lk}(c_1, c_2) \in \mathbf{Q}$ in M by

$$\mathrm{lk}(c_1, c_2) = \frac{1}{m}(\tilde{c}_1 \cdot c_2),$$

where $\tilde{c}_1 \cdot c_2$ denotes the intersection number in M. It is easy to check that this is independent of the choice of m or $\tilde{c}_1$ and that it is skew-symmetric.

Now for $\alpha, \beta \in H_2(F)$, we define

$$\Gamma_K(\alpha, \beta) = \mathrm{lk}(i_* \alpha, \beta) \in \mathbf{Q},$$

where $i : F \to M \setminus F$ is the map defined by the translation in the positive normal direction (determined by the orientation of S^1). The bilinear form Γ_K is called the *rational Seifert form* of the open book (M, K, ϕ) (see also [15]). Furthermore, a matrix representative of Γ_K is called a *rational Seifert matrix* of (M, K, ϕ). It is easy to see that the above rational linking number does not depend on the choice of the 2-cycles representing the relevant homology classes. Here we note that $H_2(F)$ is always a finitely generated free abelian group, since F is 1-connected.

Remark 2.3. Let

$$b_M : H_2(M) \times H_2(M) \to \mathbf{Q}/\mathbf{Z}$$

be the *torsion linking form* of the 5-manifold M (see [2, §§0.7 and 0.8]). Then we naturally have the commutative diagram

$$\begin{array}{ccc} H_2(F) \times H_2(F) & \xrightarrow{\Gamma_K} & \mathbf{Q} \\ \Big\downarrow{\scriptstyle i_{F*} \times i_{F*}} & & \Big\downarrow{\scriptstyle \pi} \\ H_2(M) \times H_2(M) & \xrightarrow{b_M} & \mathbf{Q}/\mathbf{Z}, \end{array}$$

where $i_F : F \to M$ is the inclusion map and π is the natural projection.

Consider the smooth fibration ϕ restricted to the exterior of K, $\phi : S^5 \setminus \mathrm{Int}\, N(K) \to S^1$, where we identify the fibers with F. Then a *geometric monodromy* $h : F \to F$ is defined up to isotopy relative to boundary; i.e., the total space is obtained from $F \times [0, 1]$ by gluing $F \times \{1\}$ and $F \times \{0\}$ by the diffeomorphism h. Note that $h|_{\partial F}$ is the identity map by Definition 2.1. Then we define the *variation map* $\Delta_K : H_2(F, \partial F) \to H_2(F)$ as follows (see also [7]). For a homology class $\gamma \in H_2(F, \partial F)$, take a 2-cycle $(D, \partial D)$ in $(F, \partial F)$ representing γ. Then $D \cup (-h(D))$ is a 2-cycle in F and we define $\Delta_K(\gamma)$ to be the class represented by $D \cup (-h(D))$. Note that this does not depend on the choice of $(D, \partial D)$ nor on h.

Let $\{\alpha_1, \alpha_2, \dots, \alpha_r\}$ be a base of $H_2(F)$ over the integers, where $r = \mathrm{rank}\, H_2(F)$. By Poincaré duality, there exists a base $\{\alpha_1^*, \alpha_2^*, \dots, \alpha_r^*\}$ of $H_2(F, \partial F)$ such that the intersection number in F satisfies $\alpha_k \cdot \alpha_l^* = \delta_{kl}$, where

$$\delta_{kl} = \begin{cases} 1, & k = l, \\ 0, & k \neq l. \end{cases}$$

Lemma 2.4. *Let Γ and Δ be the matrix representatives of the rational Seifert form Γ_K and the variation map Δ_K respectively with respect to the above bases. Then we have $\Gamma\Delta = E$, where E is the identity matrix.*

Proof. By using an argument similar to that in [7, §2], we see that

$$\begin{aligned}\Gamma_K(\alpha_k, \Delta_K(\alpha_l^*)) &= \mathrm{lk}(i_*\alpha_k, \Delta_K(\alpha_l^*)) \\ &= \mathrm{lk}(i_*\alpha_k, \alpha_l^* - h_*\alpha_l^*) \\ &= \mathrm{lk}(\alpha_k, (i_-)_*\alpha_l^* - i_*\alpha_l^*) \\ &= -\mathrm{lk}((i_-)_*\alpha_l^* - i_*\alpha_l^*, \alpha_k) \\ &= \alpha_k \cdot \alpha_l^* = \delta_{kl},\end{aligned}$$

where $i_- : F \to M \backslash F$ denotes the translation in the negative normal direction. Then the result follows immediately. □

Remark 2.5. Since $H_3(M) = 0$, we have the exact sequence

$$0 \longrightarrow H_2(F, \partial F) \xrightarrow{\Delta_K} H_2(F) \xrightarrow{i_{F*}} H_2(M) \longrightarrow 0 \tag{2.1}$$

(for details, see [7, §3], for example.) This shows that the determinant of Δ_K is equal to $\pm|H_2(M)|$, where $|H_2(M)|$ denotes the order of the finite abelian group $H_2(M)$. Thus by the above lemma, we see that the determinant of a rational Seifert matrix is equal to $\pm 1/|H_2(M)|$.

Let us give four notions of equivalence for open books on a 5-dimensional manifold.

Definition 2.6. Let (M, K_j, ϕ_j), $j = 0, 1$, be two simple open books on a given 5-dimensional manifold M.

(1) We say that (M, K_j, ϕ_j) have *topologically equivalent bindings*, if there exists an orientation preserving homeomorphism $\Phi : M \to M$ which maps K_0 onto K_1 preserving the orientations.

(2) We say that (M, K_j, ϕ_j) have *smoothly equivalent bindings*, if Φ can be chosen as a diffeomorphism in (1).

(3) We say that (M, K_j, ϕ_j) are *topologically equivalent*, if there exists an orientation preserving homeomorphism $\Phi : M \to M$ which maps K_0 onto K_1 preserving the orientations and which makes the following diagram commutative:

$$\begin{array}{ccc} M\backslash K_0 & \xrightarrow{\Phi} & M\backslash K_1 \\ & {\scriptstyle\phi_0}\searrow \quad \swarrow{\scriptstyle\phi_1} & \\ & S^1. & \end{array}$$

(4) We say that (M, K_j, ϕ_j) are *smoothly equivalent*, if Φ can be chosen as a diffeomorphism in (3).

As has been observed in [22], it is very difficult to classify open books on 5-dimensional manifolds up to smooth equivalence, mainly because of the difficulty in dealing with smooth structures on 4-dimensional manifolds.

The main result of this section is the following, which concerns the other three equivalences.

Theorem 2.7. *Let M be a simply connected rational homology* 5-*sphere and let (M, K_j, ϕ_j), $j = 0, 1$, be simple open books with pages F_j. Then the following five are equivalent.*

(1) *(M, K_j, ϕ_j) have smoothly equivalent bindings.*
(2) *(M, K_j, ϕ_j) have topologically equivalent bindings.*
(3) *(M, K_j, ϕ_j) are topologically equivalent.*
(4) *There exists an orientation preserving homeomorphism $\Psi : F_0 \to F_1$ which makes the following diagram commutative:*

$$\begin{array}{ccccc} H_2(F_0) & \times & H_2(F_0) & \xrightarrow{\Gamma_{K_0}} & \mathbf{Q} \\ \Psi_* \downarrow & & \Psi_* \downarrow & & \downarrow \mathrm{id} \\ H_2(F_1) & \times & H_2(F_1) & \xrightarrow{\Gamma_{K_1}} & \mathbf{Q}. \end{array}$$

(5) *There exists an orientation preserving homeomorphism $\Psi : F_0 \to F_1$ which makes the following diagram commutative:*

$$\begin{array}{ccc} H_2(F_0, \partial F_0) & \xrightarrow{\Delta_{K_0}} & H_2(F_0) \\ \Psi_* \downarrow & & \downarrow \Psi_* \\ H_2(F_1, \partial F_1) & \xrightarrow{\Delta_{K_1}} & H_2(F_1). \end{array}$$

To prove Theorem 2.7, we need the following lemma, which has been proved in [22, §2].

Lemma 2.8. *Let F be a smooth compact* 1-*connected* 4-*manifold with connected boundary. Then for some nonnegative integer k, $F \sharp k(S^2 \times S^2)$ admits a handlebody decomposition without* 3-*handles.*

Proof of Theorem 2.7. (1) $\Rightarrow$ (2): This is obvious.

(2) $\Rightarrow$ (3): This can be proved by exactly the same argument as in [22, Theorem 5.1].

(3) $\Rightarrow$ (4): This is obvious from the definition of rational Seifert forms.

(4) $\Leftrightarrow$ (5): This is obvious in view of Lemma 2.4 which implies that giving the rational Seifert form is equivalent to giving the variation map.

(4) $\Rightarrow$ (1): Note that the condition (5) also holds for the same homeo-

morphism Ψ. By the exact sequence (2.1), we see that there exists an isomorphism $\eta : H_2(M) \to H_2(M)$ which makes the following diagram commutative:

$$\begin{array}{ccc} H_2(F_0) & \xrightarrow{\Psi_*} & H_2(F_1) \\ \big\downarrow{\scriptstyle i_{F_0 *}} & & \big\downarrow{\scriptstyle i_{F_1 *}} \\ H_2(M) & \xrightarrow{\eta} & H_2(M), \end{array}$$

where $i_{F_j} : F_j \to M$, $j = 0, 1$, denote the inclusions. The homeomorphism Ψ preserves the second Stiefel-Whitney classes of F_j, which implies that η preserves the second Stiefel-Whitney class of M as well. Furthermore, by the condition (4) together with Remark 2.3, we see that η also preserves the torsion linking form of M. Hence by Barden [2, Theorem 2.2], there exists an orientation preserving diffeomorphism $\Phi_1 : M \to M$ such that $\Phi_{1*} = \eta$. Thus, by using the diffeomorphism Φ_1, we may assume that the diagram

$$\begin{array}{ccc} H_2(F_0) & \xrightarrow{\Psi_*} & H_2(F_1) \\ & {\scriptstyle i_{F_0 *}} \searrow \quad \swarrow {\scriptstyle i_{F_1 *}} & \\ & H_2(M) & \end{array}$$

commutes.

On the other hand, since $\Psi : F_0 \to F_1$ is a homeomorphism, by an argument of Boyer [3, p. 347], we see that there exists a smooth 5-dimensional h-cobordism W between F_0 and F_1 such that the composition

$$H_2(F_0) \xrightarrow{\iota_{0*}} H_2(W) \xrightarrow{(\iota_{1*})^{-1}} H_2(F_1)$$

coincides with Ψ_*, where $\iota_j : F_j \to W$ are the inclusion maps. Then by the stable h-cobordism theorem [13], [18], $W \sharp_c k(S^2 \times S^2 \times I)$ is diffeomorphic to the product $(F_0 \sharp k(S^2 \times S^2)) \times I$ for some nonnegative integer k, where $I = [0, 1]$ and $\sharp_c$ denotes a connected sum along cobordisms. Hence there exists a diffeomorphism $\tilde{\Psi} : F_0 \sharp k(S^2 \times S^2) \to F_1 \sharp k(S^2 \times S^2)$ such that $\tilde{\Psi}_* = \Psi_* \oplus \mathrm{id}$ with respect to the decomposition $H_2(F_j \sharp k(S^2 \times S^2)) \cong H_2(F_j) \oplus H_2(\sharp^k(S^2 \times S^2))$. Note that we can embed $S^2 \times S^2$ in a 5-disk in M so that we can perform the connected sum operation of F_j with $S^2 \times S^2$ in M. Thus we obtain Seifert manifolds $\tilde{F}_j \cong F_j \sharp k(S^2 \times S^2)$ of K_j and a diffeomorphism $\tilde{\Psi} : \tilde{F}_0 \to \tilde{F}_1$ which preserves the rational Seifert forms. By Lemma 2.8 we may further assume that $\tilde{F}_j$ have handlebody decompositions without 3-handles.

By using the argument in [14, § 18], we see that there exist decompositions

$$\tilde{F}_j = \Sigma_j \cup h^2_{1,j} \cup h^2_{2,j} \cup \cdots \cup h^2_{r,j}, \qquad j = 0, 1,$$

preserved by $\tilde{\Psi}$, where Σ_j is a homology 4-ball and $h^2_{l,j}$ are 2-handles attached to Σ_j along the boundary simultaneously. Then the diffeomorphism between the regular neighborhoods of Σ_0 and Σ_1 in M induced by $\tilde{\Psi}|_{\Sigma_0}$ extends to a diffeomorphism Φ_2 of M as follows. Let M_j be the closure of the complement of the regular neighborhood of Σ_j in M. Note that M_j is a compact simply connected 5-manifold with boundary and we have a diffeomorphism $\varphi : \partial M_0 \to \partial M_1$ which is induced by the diffeomorphism between the regular neighborhoods of Σ_j. Then by [2, Theorem 2.2], we see that the closed 5-manifold $M_0 \cup_\varphi (-M_1)$, which is obtained by gluing M_0 and $-M_1$ along their boundaries by using φ, is diffeomorphic to the connected sum $M\sharp(-M)$. Thus $M_0 \cup_\varphi (-M_1)$ bounds a compact 6-manifold X diffeomorphic to $(M \backslash \mathrm{Int}\, D^5) \times [0,1]$. Then we can easily show that X is an h-cobordism relative to boundary between M_0 and M_1. Hence, by the h-cobordism theorem [24], we see that the diffeomorphism $\varphi : \partial M_0 \to \partial M_1$ extends to a diffeomorphism $M_0 \to M_1$ which induces the "identity" on homology. Therefore, there exists an orientation preserving diffeomorphism $\Phi_2 : M \to M$ such that $\Phi_2(\Sigma_0) = \Sigma_1$ and $\Phi_{2*} : H_*(M) \to H_*(M)$ is the identity map. Note that now we have the following commutative diagram:

$$\begin{array}{ccc} H_2(F_0\sharp k(S^2 \times S^2)) & \xrightarrow{\tilde{\Psi}_*} & H_2(F_1\sharp k(S^2 \times S^2)) \\ \Big\downarrow{\scriptstyle \tilde{i}_{0*}} & & \Big\downarrow{\scriptstyle \tilde{i}_{1*}} \\ H_2(M) & \xrightarrow{\Phi_{2*}} & H_2(M), \end{array}$$

where $\tilde{i}_j : F_j\sharp k(S^2 \times S^2) \to M$, $j = 0, 1$, denote the inclusion maps.

Then by using an argument similar to that in [14, §20], we can smoothly isotope $\Phi_2(\tilde{F}_0)$ to $\tilde{F}_1$ in M so that the induced diffeomorphism $\tilde{F}_0 \to \tilde{F}_1$ coincides with $\tilde{\Psi}$. Thus, by using the isotopy extension theorem, we obtain a desired diffeomorphism of M, since $\partial\tilde{F}_j = \partial(F_j\sharp k(S^2 \times S^2)) = K_j$, $j = 0, 1$.

This completes the proof. □

REMARK 2.9. In the proof of Theorem 2.7 (4) $\Rightarrow$ (1), let us assume that $F_0 = F_1$ as abstract 4-manifolds, that $\Psi = \mathrm{id}$ and that the diagram

$$\begin{array}{ccc} H_2(F_0) & \xrightarrow{\Psi_*} & H_2(F_1) \\ & {\scriptstyle i_{F_0*}} \searrow \quad \swarrow {\scriptstyle i_{F_1*}} & \\ & H_2(M) & \end{array}$$

commutes. Then, as the above proof shows, the diffeomorphism Φ can be chosen so that $\Phi|_{F_0} = \mathrm{id}$. See also [22, Lemma 8.3].

REMARK 2.10. Let K be a smoothly embedded closed connected oriented

3-manifold in M with trivial normal bundle. If there exists a topological (not necessarily smooth) fibration $\phi : M \setminus K \to S^1$ satisfying the condition in Definition 2.1 and with simply connected fibers such that at least one of the fibers of ϕ is smooth and smoothly embedded in M, then we call (M, K, ϕ) a simple *almost* open book. Then Theorem 2.7 holds also for simple almost open books.

Remark 2.11. In Theorem 2.7 (4) and (5), we cannot replace the homeomorphism $\Psi : F_0 \to F_1$ with a diffeomorphism in general. Also in (3), we cannot replace topological equivalence with smooth equivalence. In fact, there exist simple open books (S^5, K_j, ϕ_j), $j = 0, 1$, such that they are topologically equivalent, but that their pages are not diffeomorphic to each other. See [22, § 5] for details.

Remark 2.12. We can show that each of the conditions of Theorem 2.7 is equivalent to the following:

(6) The pages of (M, K_j, ϕ_j) are orientation preservingly homeomorphic and the geometric monodromies are homotopic relative to boundary.

In fact, we see easily that (3) implies (6). Conversely, suppose that (6) holds. Then by [19], the geometric monodromies of (M, K_j, ϕ_j) are topologically isotopic relative to boundary. Thus (3) holds.

Compare Theorem 2.7 and Remark 2.12 with [22, Theorem 2.2] and [22, Corollary 2.9].

Remark 2.13. By using a result of Boyer [3], we see that the geometric conditions (4) and (5) of Theorem 2.7 can be replaced by the following more algebraic conditions (4)′ and (5)′ respectively.

(4)′ There exist an orientation preserving homeomorphism $\psi : K_0 \to K_1$ and an isomorphism $\Lambda : H_2(F_0) \to H_2(F_1)$ which preserves the rational Seifert forms such that

(a) the diagram

$$\begin{array}{ccccccccccc} 0 & \longrightarrow & H_2(K_0) & \longrightarrow & H_2(F_0) & \longrightarrow & H_2(F_0, K_0) & \longrightarrow & H_1(K_1) & \longrightarrow & 0 \\ & & \downarrow{\scriptstyle \psi_*} & & \downarrow{\scriptstyle \Lambda} & & \uparrow{\scriptstyle \Lambda^*} & & \downarrow{\scriptstyle \psi_*} & & \\ 0 & \longrightarrow & H_2(K_1) & \longrightarrow & H_2(F_1) & \longrightarrow & H_2(F_1, K_1) & \longrightarrow & H_1(K_1) & \longrightarrow & 0 \end{array}$$

commutes, where the two horizontal sequences are the exact sequences of the pairs (F_0, K_0) and (F_1, K_1) respectively, and Λ^* is the adjoint of Λ with respect to the identification of $H_2(F_j, \partial F_j)$ with $\mathrm{Hom}(H_2(F_j), \mathbf{Z})$ arising from the Poincaré-Lefschetz duality,

(b) the obstruction $\theta(\psi, \Lambda)$ defined by Boyer [3] vanishes.

(5)′ There exist an orientation preserving homeomorphism $\psi : K_0 \to K_1$ and an isomorphism $\Lambda : H_2(F_0) \to H_2(F_1)$ such that the diagram

$$\begin{array}{ccc} H_2(F_0, K_0) & \xrightarrow{\Delta_{K_0}} & H_2(F_0) \\ {\scriptstyle \Lambda^*} \big\uparrow & & \big\downarrow {\scriptstyle \Lambda} \\ H_2(F_1, K_1) & \xrightarrow{\Delta_{K_1}} & H_2(F_1) \end{array}$$

commutes and that the above conditions (a) and (b) are satisfied.

It is known that if $H_*(K_j; \mathbf{Q}) = 0$, then Boyer's obstruction always vanishes (see [3, (0.8) Proposition]). Thus as in [22, Theorem 2.12], we can show that if K_j are orientation preservingly diffeomorphic to a 3-manifold Σ such that $H_1(\Sigma) \cong \mathbf{Z}/m\mathbf{Z}$, where $m = 1, 2, 4, p^n$ or $2p^n$ with p being an odd prime, then the two open books (M, K_j, ϕ_j) are topologically equivalent if and only if their rational Seifert forms are algebraically isomorphic.

3. Stable equivalence

In Theorem 2.7, we have used topological equivalence of open books, and as has been remarked in Remark 2.11, we cannot replace this with smooth equivalence in general. In this section, we will consider such stronger equivalence, but under the condition that we allow ourselves to stabilize open books.

First let us recall the notion of a stabilizer. In [21], it has been shown that there exists a simple open book (S^5, K_S, ϕ_S), called a *stabilizer*, such that K_S is diffeomorphic to the standard 3-sphere S^3 and that the page is diffeomorphic to $(S^2 \times S^2) \sharp (S^2 \times S^2) \setminus \mathrm{Int}\, D^4$. Such an open book is not unique, and we fix one once for all in the following argument.

Recall that given two simple open books (M_j, K_j, ϕ_j), $j = 0, 1$, with $\dim M_0 = \dim M_1$, we can define their *connected sum* $(M_0, K_0, \phi_0) \sharp (M_1, K_1, \phi_1)$, which is also a simple open book.

Definition 3.1. Let (M, K_j, ϕ_j), $j = 0, 1$, be simple open books on a given 5-dimensional manifold M. We say that (M, K_j, ϕ_j) are *stably smoothly equivalent* (or *stably equivalent* for short), if $(M, K_j, \phi_j) \sharp k(S^5, K_S, \phi_S)$ are smoothly equivalent for some nonnegative integer k, where $(M, K_j, \phi_j) \sharp k(S^5, K_S, \phi_S)$ denotes the simple open book obtained by the connected sum of (M, K_j, ϕ_j) with k copies of the stabilizer (S^5, K_S, ϕ_S).

Then by using the same argument as in the proof of [22, Proposition 5.7], we can prove the following.

Proposition 3.2. *Let M be a simply connected rational homology 5-sphere* *and* (M, K_j, ϕ_j), $j = 0, 1$, *be simple open books. If* (M, K_j, ϕ_j) *are topologically equivalent, or if they have smoothly equivalent bindings, then they are stably equivalent.*

In other words, each of the five conditions in Theorem 2.7 implies that the open books are stably equivalent. We do not know if the converse is true or not.

4. Stable realization of a given page

In this section, we realize a given 4-manifold as the page of some open book up to taking connected sum with some copies of $S^2 \times S^2$. The ambient 5-manifold will be the 5-sphere if the 4-manifold is spin; otherwise, it will be the simplest nonspin 1-connected 5-manifold which is described as follows.

Let X_{-1} be the unique closed 1-connected 5-manifold with $H_2(X_{-1}) \cong \mathbf{Z}_2$ (see [2]). This manifold X_{-1} is obtained, for example, as $S^2 \tilde{\times} D^3 \cup_\varphi S^2 \tilde{\times} D^3$, where $S^2 \tilde{\times} D^3$ is the nontrivial D^3-bundle over S^2 with $\partial(S^2 \tilde{\times} D^3) \cong \mathbf{C}P^2 \sharp (-\mathbf{C}P^2)$ and φ is a self-diffeomorphism of $\mathbf{C}P^2 \sharp (-\mathbf{C}P^2)$ inducing the automorphism represented by the matrix

$$\begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$$

on the second homology with respect to the base $\{[\mathbf{C}P_1^1], [\mathbf{C}P_2^1]\}$ represented by $\mathbf{C}P_1^1 \subset \mathbf{C}P^2$ and $\mathbf{C}P_2^1 \subset -\mathbf{C}P^2$. Note that X_{-1} is not spin; i.e., its second Stiefel-Whitney class $w_2(X_{-1}) \in H^2(X_{-1}; \mathbf{Z}_2)$ does not vanish. Note also that the torsion linking form

$$b_{X_{-1}} : H_2(X_{-1}) \times H_2(X_{-1}) \to \mathbf{Q}/\mathbf{Z}$$

is given by $b_{X_{-1}}(\alpha, \alpha) \equiv 1/2 \pmod 1$, where $\alpha \in H_2(X_{-1}) \cong \mathbf{Z}_2$ is the generator.

The main result of this section is the following.

THEOREM 4.1. *Let* F *be a smooth compact* 1*-connected* 4*-manifold with nonempty connected boundary.*

(1) *If* F *is spin, then there exists a simple open book on* S^5 *whose page is diffeomorphic to* $F \sharp k(S^2 \times S^2)$ *for some nonnegative integer* k.
(2) *If* F *is not spin, then there exists a simple open book on* X_{-1} *whose page is diffeomorphic to* $F \sharp k(S^2 \times S^2)$ *for some nonnegative integer* k.

PROOF. Let us first prove the part (2). Since $H_1(F)$ has no 2-torsion, every element of $H^2(F; \mathbf{Z}_2)$ or $H^2(F, \partial F; \mathbf{Z}_2)$ has an integral lift. Furthermore, the second Stiefel-Whitney class $w_2(F) \in H^2(F; \mathbf{Z}_2)$ of F is characterized by the property

$$\langle w_2(F) \smile \gamma, [F, \partial F]_2 \rangle = \langle \gamma \smile \gamma, [F, \partial F]_2 \rangle$$

for all $\gamma \in H^2(F, \partial F; \mathbf{Z}_2)$, where $[F, \partial F]_2 \in H_4(F, \partial F; \mathbf{Z}_2)$ denotes the $\mathbf{Z}_2$ fundamental class. Since F is not spin and $w_2(F) \neq 0$, the above observation shows that there exists a primitive homology class $\alpha \in H_2(F)$ such that $\alpha \cdot \alpha$ is odd. Furthermore, by taking connected sum with $S^2 \times S^2$, we may assume that there exist elements $\beta_\pm \in H_2(F)$ such that $\beta_\pm \cdot \beta_\pm = \pm 2$ and that $\alpha \cdot \beta_\pm = 0$. Using α and $\beta_\pm$, it is not difficult to construct a base $\{\alpha_1, \alpha_2, \ldots, \alpha_r\}$ of $H_2(F)$ such that $\alpha_1 \cdot \alpha_1 = \pm 1$ and $\alpha_l \cdot \alpha_l$ are even for all $l \geq 2$.

Let us denote $F \sharp k(S^2 \times S^2)$ by F_k. Furthermore, let Q_k be the intersection matrix of F_k with respect to a base of $H_2(F_k) \cong H_2(F) \oplus H_2(\sharp^k S^2 \times S^2)$ which extends the base $\{\alpha_1, \alpha_2, \ldots, \alpha_r\}$ of $H_2(F)$. Then by using a method similar to [20, §6], we can prove the following.

Lemma 4.2. *For some nonnegative integer k, there exists a rational square matrix L with the following properties.*

(1) *$L + {}^tL$ coincides with the intersection matrix Q_k of F_k.*

(2) $\det L = \pm 1/2$.

(3) *One of the diagonal entries of L is equal to $\pm 1/2$ and all the other entries of L are integers.*

Proof. It is easy to see that there exists an $r \times r$ triangular matrix L_1 such that $L_1 + {}^tL_1$ coincides with the intersection matrix of F and that all the entries of L_1 are integers except exactly the first diagonal entry which is equal to $\pm 1/2$, where $r = \operatorname{rank} H_2(F)$. Set $k = r - 1$. Then the $(r + 2k) \times (r + 2k)$ matrix

$$L = \begin{pmatrix} L_1 & R_2 & R_3 & \cdots & R_r \\ -{}^tR_2 & U & & & \\ -{}^tR_3 & & U & 0 & \\ \vdots & & 0 & \ddots & \\ -{}^tR_r & & & & U \end{pmatrix}$$

satisfies the required properties, where R_l is the $r \times 2$ matrix $(x_{ij}^{(l)})_{1 \leq i \leq r, 1 \leq j \leq 2}$ such that

$$x_{ij}^{(l)} = \begin{cases} 0, & i \neq l, \\ 1, & i = l, \end{cases}$$

and

$$U = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}.$$

This completes the proof of Lemma 4.2. □

By Lemma 2.8, we may further assume that F_k has a handlebody decomposition without 3-handles. Let $F_k = \Sigma \cup h_1^2 \cup h_2^2 \cup \cdots \cup h_{r+2k}^2$ be a decomposition as in §2, where Σ is a homology 4-ball and h_l^2 are 2-handles attached to Σ along the boundary simultaneously. We may assume that the 2-handles correspond to the base of $H_2(F_k)$ with respect to which Q_k is given. Furthermore, we may assume that the $(1,1)$-entry of L is equal to $\pm 1/2$. Then the value of $w_2(F_k)$ evaluated on the $\mathbf{Z}_2$ homology class corresponding to h_l^2 is zero for $l \geq 2$ and is nonzero for $l = 1$.

Lemma 4.3. *Let k and L be as above. Then there exists an embedding $\zeta : F_k \hookrightarrow X_{-1}$ whose rational Seifert matrix is given by L.*

Proof. Since Σ consists only of 0-, 1-, and 2-handles, we can embed Σ in a 5-disk in X_{-1}. Then we can extend the embedding to an embedding θ of the union of Σ with the cores of the 2-handles h_l^2 so that the core of h_1^2 corresponds to the generator of $H_2(X_{-1}) \cong \mathbf{Z}_2$ and that the others correspond to the zero homology class. We can further extend this embedding to an embedding of the whole F_k into X_{-1}, using the fact that the pull-back of $w_2(X_{-1})$ by θ coincides with $w_2(F_k)$.

Finally, by using a method of Kervaire [8, pp. 255–257], we can adjust the rational Seifert form. Here we have to be careful, since the entries of the matrix L are not necessarily integers. However, by using Remark 2.3, we see that the necessary adjustments are always integral, so that Kervaire's method works without any problem. □

By the above lemma, we identify F_k with $\zeta(F_k)$ in X_{-1}, and denote by $i' : F_k \to X_{-1}$ the inclusion. Set $W = X_{-1} \setminus \operatorname{Int} N(F_k)$, where $N(F_k)$ is a tubular neighborhood of F_k in X_{-1}. Note that W is a smooth 5-dimensional cobordism relative to boundary between two copies of F_k.

Since X_{-1} is simply connected and F_k has a handlebody decomposition without 3- or 4-handles, W is also simply connected. Furthermore, by using standard homology and cohomology arguments together with the fact that $i'_* : H_2(F_k) \to H_2(X_{-1})$ is surjective, we can show that $H_*(W) \cong H_*(F_k)$ and that the inclusion $i_W : W \to X_{-1}$ induces an epimorphism $i_{W*} : H_2(W) \to H_2(X_{-1})$.

By the Künneth theorem, we have $H_2(F_k, \partial F_k) \cong H_3(F_k \times [0,1], \partial(F_k \times [0,1])) \cong H_3(N(F_k), \partial N(F_k))$, which in turn is isomorphic to $H_3(X_{-1}, W)$ by excision. Furthermore, by the exact sequence

$$0 \longrightarrow H_3(X_{-1}, W) \xrightarrow{\partial} H_2(W) \xrightarrow{i_{W*}} H_2(X_{-1})$$

of the pair (X_{-1}, W), we see that $\partial : H_3(X_{-1}, W) \to \ker i_{W*}$ is an isomorphism. Thus $H_2(F_k, \partial F_k)$ is naturally isomorphic to $\ker i_{W*}$.

On the other hand, since $i_{W*} : H_2(W) \to H_2(X_{-1})$ is an epimorphism, we have the exact sequence

$$0 \longrightarrow \ker i_{W*} \longrightarrow H_2(W) \xrightarrow{i_{W*}} H_2(X_{-1}) \longrightarrow 0.$$

Thus, there exists a base $\{\tilde{\beta}_1, \tilde{\beta}_2, \dots, \tilde{\beta}_s\}$ of $H_2(W)$ $(s = r + 2k)$ such that $\{\tilde{B}_i = r_i\tilde{\beta}_i\}_{1 \le i \le s}$ forms a base of $\ker i_{W*}$ for some integers $r_1, r_2, \dots, r_s$ with $r_1 r_2 \cdots r_s = 2 = |H_2(X_{-1})|$. Let $\{B_i\}_{1 \le i \le s}$ be the base of $H_2(F_k, \partial F_k)$ corresponding to $\{\tilde{B}_i\}$ by the isomorphism described above. Furthermore, let $\{\beta_i\}_{1 \le i \le s}$ be the base of $H_2(F_k)$ dual to $\{B_i\}$ with respect to the intersection form.

Let $i_+ : F_k \to W$ denote the inclusion map into the positive side. We denote by $Y = (y_{ij})$ the matrix representative of $i_{+*} : H_2(F_k) \to H_2(W)$ with respect to the above bases. We denote the rational Seifert form of F_k in X_{-1} by Γ_{F_k}. Then we have

$$\begin{aligned}
\Gamma_{F_k}(\beta_i, \beta_j) &= \mathrm{lk}(i_{+*}\beta_i, \beta_j) = \mathrm{lk}\left(\sum_{l=1}^{s} y_{li}\tilde{\beta}_l, \beta_j\right) \\
&= \sum_{l=1}^{s} y_{li}\, \mathrm{lk}(\tilde{\beta}_l, \beta_j) = \sum_{l=1}^{s} \frac{1}{r_l} y_{li}\, \mathrm{lk}(\tilde{B}_l, \beta_j) \\
&= \pm \sum_{l=1}^{s} \frac{1}{r_l} y_{li} \beta_j \cdot B_l \qquad (4.1) \\
&= \pm \sum_{l=1}^{s} \frac{1}{r_l} y_{li} \delta_{jl} = \pm \frac{1}{r_j} y_{ji},
\end{aligned}$$

where (4.1) follows from the definition of the linking number in X_{-1} together with the construction of the isomorphism $H_2(F_k, \partial F_k) \cong \ker i_{W*}$. Thus the determinant of Γ_{F_k} is equal to the determinant of i_{+*} divided by $r_1 r_2 \cdots r_s = 2$. Since the determinant of the rational Seifert form Γ_{F_k} is equal to $\pm |H_2(X_{-1})|^{-1} = \pm 1/2$ by Lemma 4.2 (2), we see that $i_{+*} : H_2(F_k) \to H_2(W)$ is an isomorphism. By a similar argument, we can show that $i_{-*} : H_2(F_k) \to H_2(W)$ is also an isomorphism, where $i_- : F_k \to W$ is the inclusion into the negative side. Thus W is an h-cobordism relative to boundary (this type of argument is almost identical with [15, Ch. 4]). Then by applying the stabilization construction [21, Proposition 4.4] to the embedding constructed in the above lemma, we get the desired open book on X_{-1}.

For the part (1), use a similar argument together with the results and methods in [20, §6]. This completes the proof of Theorem 4.1. □

5. Isotopy of 1-connected 4-manifolds with boundary

It has been shown in [19] that two (orientation preserving) homeomorphisms of a *closed* simply connected 4-manifold are homotopic if and only if the induced automorphisms on the second homology group coincide with each other (see also [4, § 5]). Using this, Quinn [19] has shown that two homeomorphisms inducing the same automorphism on the second homology group are actually topologically isotopic (see also [16]). Quinn has also shown that two diffeomorphisms inducing the same automorphism are smoothly isotopic after the connected sum with the identity diffeomorphism of some copies of $S^2 \times S^2$. In this section we apply our results in the previous sections to get a similar stable isotopy criterion for the case where the simply connected 4-manifolds have nonempty connected boundaries.

Definition 5.1. Let F be a compact 4-manifold with boundary and $h_0, h_1 : F \to F$ two diffeomorphisms with $h_0|_{\partial F} = h_1|_{\partial F}$ being the identity map. We say that h_0 and h_1 are *stably isotopic* relative to boundary if $h_0 \sharp k(\mathrm{id})$ and $h_1 \sharp k(\mathrm{id})$ are smoothly isotopic relative to boundary as diffeomorphisms of $F_k = F \sharp k(S^2 \times S^2)$ for some k.

Suppose that $h : F \to F$ is a diffeomorphism which is the identity on the boundary. Then we can define the *variation map* $\Delta_h : H_2(F, \partial F) \to H_2(F)$ exactly as in § 2 (see the paragraph just after Remark 2.3). Note that if two such diffeomorphisms are isotopic relative to boundary, then their variation maps coincide with each other. The main result of this section is that the converse is also true "stably" as follows.

Theorem 5.2. *Let F be a compact* 1*-connected* 4*-manifold with connected and nonempty boundary. Suppose that h_0 and $h_1 : F \to F$ are two diffeomorphisms such that $h_0|_{\partial F} = h_1|_{\partial F}$ is the identity map. Then h_0 and h_1 are stably isotopic relative to boundary if and only if $\Delta_{h_0} = \Delta_{h_1} : H_2(F, \partial F) \to H_2(F)$.*

Proof. If h_0 and h_1 are stably isotopic relative to boundary, then it is easy to see that their variation maps coincide with each other, since on the direct summand of $H_2(F_k, \partial F_k)$ corresponding to $H_2(\sharp^k S^2 \times S^2)$, the variation maps of $h_j \sharp k(\mathrm{id})$, $j = 0, 1$, are the zero homomorphisms.

In order to prove the converse, let us first consider the following construction.

Let (M, K, ϕ) be the open book as in Theorem 4.1 constructed for F and let $h : F_k \to F_k$ be its geometric monodromy. Furthermore, let (M', K', ϕ') be another open book such that its page is F_k and its geometric monodromy is $(h_0 \sharp k(\mathrm{id})) \circ (h_1 \sharp k(\mathrm{id}))^{-1} \circ h$. Since $\Delta_{h_0} = \Delta_{h_1}$ by our assumption, we see that

$\Delta_K = \Delta_{K'}$ (for details, see [22, §9]). Then by using the exact sequence (2.1) and Remark 2.3 together with the results in [2], we can prove the following.

Lemma 5.3. *The* 5-*manifold* M' *is diffeomorphic to* M.

Since the variation maps of the two open books coincide with each other, there exists a diffeomorphism $\Phi : M \to M'$ with $\Phi(\partial F_k) = \partial F_k$ for some k by Theorem 2.2. Furthermore, by using an argument similar to that in the proof of [22, Lemma 8.3], we may assume that $\Phi|_{F_k} = \mathrm{id}$ (see also Remark 2.9). Then we see that h and $(h_0 \sharp k(\mathrm{id})) \circ (h_1 \sharp k(\mathrm{id}))^{-1} \circ h$ are smoothly pseudo-isotopic relative to boundary by Wall's construction [26, pp. 140–141] (see also [11] or the proof of [22, Lemma 2.5]). Then it is easy to see that $h_0 \sharp k(\mathrm{id})$ and $h_1 \sharp k(\mathrm{id})$ are pseudo-isotopic relative to boundary. Then by Quinn [19], they are stably isotopic relative to boundary. This completes the proof of Theorem 5.2. □

Note that Theorem 5.2 solves [22, Conjecture 9.6] affirmatively. It should be pointed out that this has been possible here, since we have generalized results in [22] about open books on the 5-*sphere* to those on *rational homology* 5-*spheres*.

Remark 5.4. In the above situation, we can prove that $h_0 \sharp k(\mathrm{id})$ and $h_1 \sharp k(\mathrm{id})$ are pseudo-isotopic relative to boundary also by using an argument of Kreck [9, proof of Theorem 1] based on surgery techniques (see also the last paragraph of [10]). For details, see [23].

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