

## 第7回広島整数論集会

下記の日程で研究集会を開催します。

2008年7月22日(火) – 7月25日(金)

広島大学理学部 B棟 B707

本研究集会は、日本学術振興会・先端研究拠点事業「数論幾何・モチーフ理論・ガロア理論の新展開と、その社会的実用」(コーディネーター 松本眞)、科研費基盤研究(A)「数論・幾何の新展開: 数論的トポロジー、圏論的数論幾何、アルゴリズム」(代表者 松本眞)、科研費基盤研究(B)「数論的多様体の  $p$  進的手法による研究」(代表者 都築暢夫)からの補助を受けております。

プログラムの変更・講演のアブストラクト・会場までの交通手段などは、ホームページ

<http://www.math.sci.hiroshima-u.ac.jp/m-mat/JSPS-CoreToCore/SEISURON08/hiroshima08.html>

をご覧ください。

旅費を希望する方には補助できる可能性があります。都築 (tsuzuki@math.tohoku.ac.jp) まで早めに連絡して下さい。大学院生・研究生の方は、指導教員経由で申し込み下さい。

### プログラム

7月22日(火)

10:40 – 11:40 近藤 智 (東京大)/Satoshi Kondo (Tokyo)

有限体上の代数多様体の2つのモチビクコホモロジーについて (安田正大氏との共同研究)

(On two motivic cohomology groups of a variety over a finite field (joint work with Seidai Yasuda))

11:50 – 12:50 望月 哲史 (東洋大)/Satoshi Mochizuki (Toyo)

An absolute geometric presentation theorem

14:00 – 15:00 小関 祥康 (九州大)/Yoshiyasu Ozeki (Kyushu)

Torsion points of abelian varieties with values in infinite extensions over a  $p$ -adic fields

15:15 – 16:15 富山 恭敬 (九州大)/Yoshiyuki Tomiyama (Kyushu)

Lifting Galois representations over arbitrary number fields

16:30 – 17:30 今井 直毅 (東京大)/Naoki Imai (Tokyo)

On the connected components of moduli spaces of finite flat models

7月23日(水)

9:30 – 10:30 山本 修司 (東京大)/Shuji Yamamoto (Tokyo)

On Shintani's ray class invariant for totally real number fields

10:40 – 11:40 森 伸吾 (京都大)/Shingo Mori (Kyoto)

An explicit form of Gauss sums associated with the space of symmetric matrices

11:50 – 12:50 近藤 崇 (中央大)/Takashi Kondo (Chuo)

離散付値環上のある種の群スキームの拡大について  
(On the extensions of group schemes over a discrete valuation ring)

14:00 – 15:00 Valentina Di Proietto (Univ. Padova)

$p$ -adic differential equations and log-convergent isocrystals for semistable curves

15:15 – 16:15 長谷川 泰子 (東京大)/Yasuko Hasegawa (Tokyo)

Generalized principal series Whittaker functions on the real symplectic group of rank 2

16:30 – 17:30 大井 理生 (京都大)/Masao Ooi (Kyoto)

次数 2, “重み 2” のジューゲル・アイゼンシュタイン級数を対角に制限したものの Fourier 係数についての予想  
(Conjectures on the Fourier coefficients of the restriction to the diagonal of the Siegel Eisenstein series of degree 2, “weight 2”)

18:00 – 懇親会

7月24日(木)

9:30 – 10:30 大槻 玲 (慶応義塾大)/Rei Otsuki (Keio)

Construction of a homomorphism concerning Euler systems for an elliptic curve

10:40 – 11:40 原 隆 (東京大)/Takashi Hara (Tokyo)

Iwasawa theory of totally real fields for certain non-commutative  $p$ -extensions

11:50 – 12:50 津嶋 貴弘 (東京大)/Takahiro Tsushima (Tokyo)

Localized Grothendieck-Ogg-Shafarevich formula and conductor formula

14:00 – 15:00 Christopher Rasmussen (京都大数理研 (RIMS) ・ JSPS)

Finiteness results of abelian varieties with constrained torsion

15:15 – 16:15 若林 徳子 (近畿大)/Noriko Wakabayashi (Kinki)

等号付き多重ゼータ値のある和とその母関数について  
(On sums of non-strict multiple zeta values and their generating function)

16:30 – 17:30 田中 立志 (九州大)/Tatsushi Tanaka (Kyushu)

ニュートン級数と多重  $L$  値の一般導分関係式  
(Newton series and the quasi-derivation relation for multiple  $L$ -values)

7月25日(金)

9:30 – 10:30 岡野 恵司 (早稲田大)/Keiji Okano (Waseda)

The  $p$ -class field towers over the cyclotomic  $\mathbb{Z}_p$ -extensions of certain algebraic number fields

10:40 – 11:40 平之内 俊郎 (九州大)/Toshiro Hiranouchi (Kyushu)

Class field theory of curves over  $p$ -adic fields

11:50 – 12:50 中村 健太郎 (東京大)/Kentaro Nakamura (Tokyo)

Trianguline 表現の分類  
(Classification of trianguline representations)

13:50 – 14:50 内田 幸寛 (名古屋大)/Yukihiro Uchida (Nagoya)

超楕円 Jacobi 多様体の等分多項式と標準局所高さ  
(Division polynomials and canonical local heights on hyperelliptic Jacobians)

15:00 – 16:00 原下 秀士 (東京大)/Shusi Harashita (Tokyo)

Generic Newton polygons of Ekedahl-Oort strata : Oort's conjecture

世話人 : 市原由美子・松本眞 (広島大)・西来路文朗 (広島国際大)・高橋浩樹 (徳島大)・  
都築暢夫 (東北大)・山内卓也 (大阪府大)

# 第7回広島整数論集会

## アブストラクト

原 隆 (東京大)/Takashi Hara (Tokyo)

Iwasawa theory of totally real fields for certain non-commutative  $p$ -extensions

Recently, Kazuya Kato has proven the non-commutative Iwasawa main conjecture (in the sense of Coates, Fukaya, Kato, Sujatha and Venjakob) for non-commutative Galois extensions of “Heisenberg type” of totally real fields, using integral logarithmic homomorphisms introduced by Oliver and Taylor. His method was based on Burns’ outstanding idea, “construct the  $p$ -adic zeta functions for non-commutative extensions by patching  $p$ -adic zeta functions associated to commutative subextensions.” In this talk, we apply Kato’s method (and Burns’ technique) to certain non-commutative  $p$ -extensions which are more complicated than those of Heisenberg type, and sketch the proof of the main conjecture for them.

原下 秀士 (東京大)/Shusi Harashita (Tokyo)

Generic Newton polygons of Ekedahl-Oort strata : Oort’s conjecture

We study the moduli space of principally polarized abelian varieties in positive characteristic. The moduli space has two main stratifications, called Newton polygon (NP) stratification and Ekedahl-Oort (EO) stratification. It is considered very important in arithmetic geometry to clarify the singularities of NP strata. As a first step it would be natural to investigate the intersections of NP strata and EO strata. In this talk I will introduce an algorithm determining the Newton polygon of any generic point of each EO stratum, which is closely related to Oort’s conjecture on the intersections of NP strata and EO strata. I will also show that certain unions of EO strata contained in the supersingular locus can be beautifully described in terms of Deligne-Lusztig varieties.

長谷川 泰子 (東京大)/Yasuko Hasegawa (Tokyo)

Generalized principal series Whittaker functions on the real symplectic group of rank 2

We give some explicit formulas of Whittaker function belonging to the generalized principal series representations induced from the Siegel maximal parabolic subgroup on the real symplectic group of rank 2. The Whittaker function has a special  $K$ -type which are related to the work of Oda and Schwermer. Because the generalized principal series representations can be embedded in the principal series representations with respect to the minimal parabolic subgroup, we also give explicit formulas of the Whittaker function belonging to the principal series representations.

平之内 俊郎 (九州大)/Toshiro Hiranouchi (Kyushu)

Class field theory of curves over  $p$ -adic fields

We develop class field theory of curves over  $p$ -adic fields which extends the unramified theory of S. Saito (1985). The class groups which approximate abelian étale fundamental groups of such curves are introduced in the terms of algebraic  $K$ -groups by imitating G. Wiesend’s class group.

今井 直毅 (東京大)/Naoki Imai (Tokyo)

On the connected components of moduli spaces of finite flat models

Recently, Kisin made a way of getting information of a deformation ring of a local Galois representation from a moduli space of finite flat group schemes. He conjectured that the non-ordinary component of the moduli space is connected, and proved this conjecture for special  $p$ -adic fields. In this talk, we prove the conjecture for general  $p$ -adic fields. As an application, we can prove a theorem comparing a deformation ring and a Hecke ring.

近藤 智 (東京大)/Satoshi Kondo (Tokyo)

有限体上の代数多様体の2つのモチビクコホモロジーについて (安田正大氏との共同研究)

(On two motivic cohomology groups of a variety over a finite field (joint work with Seidai Yasuda))

Let  $X$  be a connected, separated, of finite type scheme over a finite field of pure dimension  $d$ . Two motivic cohomology (Borel-Moore motivic homology to be precise) groups  $H_{\mathcal{M}}^{2d+1}(X, \mathbb{Z}(d+i))$  ( $i = 1, 2$ ) of  $X$  are computed explicitly. We show that if  $X$  is not proper, then the groups are zero. If  $X$  is proper, then the group is isomorphic via the pushforward map to the groups  $H_{\mathcal{M}}^1(\text{Spec } \mathcal{O}(X), \mathbb{Z}(i))$  ( $i = 1, 2$ ) where we put  $\mathcal{O}(X) = H^0(X, \mathcal{O}_X)$ .

近藤 崇 (中央大)/Takashi Kondo (Chuo)

離散付値環上のある種の群スキームの拡大について

(On the extensions of group schemes over a discrete valuation ring)

Sekiguchi-Suwa constructed the group schemes deforming the group schemes of Witt vectors to torus for unifying Kummer theory and Artin-Schreier-Witt theory. Let  $(A, \mathfrak{m})$  be a discrete valuation ring with the maximal ideal  $\mathfrak{m}$ . Then such group schemes of dimension 1 over  $A$  are given by  $\mathcal{G}^{(\lambda)} = \text{Spec} A[X, 1/(1+\lambda X)]$  with  $\lambda \in \mathfrak{m} \setminus \{0\}$  and that is proved by Waterhouse-Weisfeiler. In higher dimensional case, Sekiguchi-Suwa decided the following successive group of extensions:

$$\begin{aligned} \text{Ext}^1(\mathcal{G}^{(\lambda_1)}, \mathcal{G}^{(\lambda_2)}) &\ni 0 \rightarrow \mathcal{G}^{(\lambda_2)} \rightarrow \mathcal{E}^{(\lambda_1, \lambda_2)} \rightarrow \mathcal{G}^{(\lambda_1)} \rightarrow 0 \\ \text{Ext}^1(\mathcal{E}^{(\lambda_1, \lambda_2)}, \mathcal{G}^{(\lambda_3)}) &\ni 0 \rightarrow \mathcal{G}^{(\lambda_3)} \rightarrow \mathcal{E}^{(\lambda_1, \lambda_2, \lambda_3)} \rightarrow \mathcal{E}^{(\lambda_1, \lambda_2)} \rightarrow 0 \\ &\dots \end{aligned}$$

In this talk, we will decide the group of extensions:  $\text{Ext}^1(\mathcal{G}^{(\xi)}, \mathcal{E}^{(\lambda_1, \lambda_2, \lambda_3)})$ . The group  $\text{Ext}^1(\mathcal{G}^{(\xi)}, \mathcal{E}^{(\lambda_1, \lambda_2)})$  was considered by Horikawa and which will be used essentially, in our argument.

望月 哲史 (東洋大)/Satoshi Mochizuki (Toyo)

An absolute geometric presentation theorem

In this lecture, I will give a survey talk about series of my papers and my progressive works which is partially jointed works with Toshiro Hiranouchi and with Akiyoshi Sannai. The main objective is how to state and prove a Noether normalization type theorem without base. In practice, if we admit a base, to study problems related with Krull (co)dimension of Modules or algebraic cycles, such a theorem might be considered as a standard tool. But I intend to emphasize that we shall say that such a theorem is not a “tool”, but rather than the core theorems in motive theory are “consequence” of such a theorem by stating the main theorem in the following way.

Absolute geometric presentation theorem: For a certain algebraic variety  $X$ , and for each integer  $p$  which is less than dimension of  $X$ , the topologically weight  $p$  part of  $X$  is weakly equivalent to motivically weight  $p$  part of  $X$ .

The difficulty of understanding the theorem is recognizing a notion of weight of algebraic geometric objects. To do so, I redefine a notion of algebraic varieties imitating Dedekind's redefining a notion of real numbers by the famous Dedekind cut. The hurdle of proving the theorem is completely divided into two different aspects. One aspect is homotopy theoretical and for example, I intend to show so-called homotopy Jordan-Holder theorem which is homotopy theoretically adaptations from original one. The other aspect is commutative algebraically theoretical and for example, I will present a notion of Koszul cubes and exhibit Iwasawa-Bourbaki-Serre type structure theorem for suitable Koszul cubes over regular local rings. In the last of my talk, I intend to explain the meaning of the theorem in the view of motive theory by turning in a new concept - motivic grammar -.

森 伸吾 (京都大)/Shingo Mori (Kyoto)

An explicit form of Gauss sums associated with the space of symmetric matrices

We think the prehomogeneous vector space of symmetric matrices over finite fields. We give an explicit form of Gauss sums associated with this space. We also give its applications to  $L$ -functions associated with the space of symmetric matrices. We show we can simplify the functional equations of  $L$ -functions.

中村 健太郎 (東京大)/Kentaro Nakamura (Tokyo)

Trianguline 表現の分類

(Classification of trianguline representations)

Trianguline representation is a class of two dimensional  $p$ -adic representations of  $p$ -adic fields. When  $p$ -adic field is  $\mathbb{Q}_p$ , this class was classified by Colmez and is very important in his study of  $p$ -adic Langlands correspondence for  $GL_2(\mathbb{Q}_p)$ . In this talk, we completely classify trianguline representations for general  $p$ -adic fields.

大井 理生 (京都大)/Masao Ooi (Kyoto)

次数 2, "重み 2" のジークル・アイゼンシュタイン級数を対角に制限したものの Fourier 係数についての予想

(Conjectures on the Fourier coefficients of the restriction to the diagonal of the Siegel Eisenstein series of degree 2, "weight 2")

Maass calculated the Fourier coefficients of the Siegel Eisenstein series of degree 2, weight  $k$  ( $k > 2$ ), level 1. In our lecture, by abuse of language, we will define the Fourier coefficients of the Siegel Eisenstein series of degree 2, "weight 2", level 1 by substituting  $k = 2$  in his formula and we give the conjectural formula of the Fourier coefficients of the restriction to the diagonal of the Siegel Eisenstein series of degree 2, "weight 2", level 1. Conjecturally, there is a norm compatible system in  $K_3$  (modular curve  $\times$  modular curve) whose image under the syntomic regulator is some trace compatible system of the restrictions to the diagonal of the Siegel Eisenstein series of degree 2, "weight 2". We explain the relation between the Fourier coefficients and the Beilinson conjecture (when time allows).

岡野 恵司 (早稲田大)/Keiji Okano (Waseda)

The  $p$ -class field towers over the cyclotomic  $\mathbb{Z}_p$ -extensions of certain algebraic number

fields

Let  $p$  be an odd prime number. We consider the maximal unramified pro- $p$ -extension of the cyclotomic  $\mathbb{Z}_p$ -extension  $F_\infty$  of a finite algebraic number field  $F$ . Using classical Iwasawa theory and the theory of central class fields, we shall find necessary conditions that the extension is abelian over  $F_\infty$ , and determine sufficient conditions in certain special cases.

大槻 玲 (慶応義塾大)/Rei Otsuki (Keio)

Construction of a homomorphism concerning Euler systems for an elliptic curve

Let  $E$  be an elliptic curve defined over  $\mathbb{Q}$ . For an odd prime  $p$ , Kurihara and Kobayashi studied Selmer groups in the cyclotomic  $\mathbb{Z}_p$ -extension of  $\mathbb{Q}(\mu_p)$  when  $E$  has supersingular reduction at  $p$ . They defined a map from local cohomology groups to group rings under which Kato's Euler system corresponds to modular element of Mazur and Tate. I will talk about a generalization of the map.

小関 祥康 (九州大)/Yoshiyasu Ozeki (Kyushu)

Torsion points of abelian varieties with values in infinite extensions over a  $p$ -adic fields

Let  $A$  be an abelian variety over a  $p$ -adic field  $K$  and  $L$  an algebraic infinite extension over  $K$ . We consider the finiteness of the torsion part of the group of rational points  $A(L)$  under some assumptions. In 1975, Hideo Imai proved that such a group is finite if  $A$  has good reduction and  $L$  is the cyclotomic  $\mathbb{Z}_p$ -extension of  $K$ . In this talk, first we show a generalization of Imai's result in the case where  $A$  has ordinary good reduction. Next we give some finiteness results when  $A$  is an elliptic curve and  $L$  is the field generated by the  $p$ -power torsion of an elliptic curve.

Valentina Di Proietto (Univ. Padova)

$p$ -adic differential equations and log-convergent isocrystals for semistable curves

We will consider a smooth and projective curve  $X_K$  over a  $p$ -adic field  $K$  and a semistable model  $X$  over a complete discrete valuation ring  $V$ , such that  $K$  is the fraction field of  $V$ . We will define a fully faithful functor between the category of log convergent isocrystals on  $X$  and the category of  $p$ -adic differential equations without singularities on  $X_K$  and we'll describe the essential image of this functor.

Christopher Rasmussen (京都大数理研 (RIMS) · JSPS)

Finiteness results of abelian varieties with constrained torsion

The most classical 'finiteness result' on abelian varieties was conjectured by Shafarevich and proven by Faltings: The number of isomorphism classes of abelian varieties with fixed dimension, field of definition, and reduction type is finite. In this talk, we discuss joint work with Akio Tamagawa on a different style of finiteness result, where the reduction type is allowed to vary in a controlled fashion. In exchange for this freedom, we place an arithmetic constraint on the structure of the pro- $p$  torsion of the abelian variety. Conjecturally, the number of isomorphism classes should still be finite. We prove this in certain cases, and explain the motivations for this conjecture in terms of Galois representations and a long-standing question of Ihara.

田中 立志 (九州大)/Tatsushi Tanaka (Kyushu)

ニュートン級数と多重  $L$  値の一般導分関係式

(Newton series and the quasi-derivation relation for multiple  $L$ -values)

Newton series is a complex function which interpolates a given sequence. We will discuss several analytic and algebraic properties of Newton series associated to truncated multiple  $L$ -values. As a result, we will obtain a class of relations for multiple  $L$ -values, which is shown to contain the quasi-derivation relation for multiple  $L$ -values conjectured by M.Kaneko. This is a joint work with G. Kawashima.

富山 恭敬 (九州大)/Yoshiyuki Tomiyama (Kyushu)

Lifting Galois representations over arbitrary number fields

In this talk, we prove that every two-dimensional residual Galois representation of the absolute Galois group of an arbitrary number field lifts to a characteristic zero  $p$ -adic representation, if local lifting problems at places above  $p$  are unobstructed. This result is a generalization of results which were proved by Ramakrishna and Gee for some number fields.

津嶋 貴弘 (東京大)/Takahiro Tsushima (Tokyo)

Localized Grothendieck-Ogg-Shafarevich formula and conductor formula

The Grothendieck-Ogg-Shafarevich formula calculates the  $l$ -adic Euler-Poincare number of an  $l$ -adic sheaf on a curve by an invariant produced by the wild ramification of the  $l$ -adic sheaf named Swan class. Recently this formula is generalized to any dimensional scheme by A.Abbes, K.Kato and T.Saito. It is formulated by the language of the characteristic class of an  $l$ -adic sheaf. In this paper, we prove the localized GOS formula assuming the strong resolution of singularities, and as an application we prove a conductor formula in equal characteristic case. The conductor formula represents the Swan conductor of an  $l$ -adic representation which appears when we consider a fibration on a curve.

内田 幸寛 (名古屋大)/Yukihiro Uchida (Nagoya)

超楕円 Jacobi 多様体の等分多項式と標準局所高さ

(Division polynomials and canonical local heights on hyperelliptic Jacobians)

In the theory of elliptic curves, the division polynomials are important for the study of the structure of the torsion subgroup. Moreover the division polynomials have various applications. For example, they appear in some equalities of the canonical local height functions. In this talk, we will define the division polynomials of hyperelliptic Jacobians over  $\mathbb{C}$  by using the hyperelliptic sigma function. And we will give some relations between the division polynomials and the canonical local height functions.

若林 徳子 (近畿大)/Noriko Wakabayashi (Kinki)

等号付き多重ゼータ値のある和とその母関数について

(On sums of non-strict multiple zeta values and their generating function)

We consider multiple zeta values defined by the Euler sums with non-strict inequalities in the summation. Generating functions for sum of (non-strict) multiple zeta values are studied by Ohno-Zagier, Aoki-Kombu-Ohno and Li. The notion of generalized height defined by Li is introduced for every natural number  $i$ . We define generating functions for sum of non-strict multiple zeta values with fixed weight, depth and generalized height and prove that the function is expressed in terms of generalized hypergeometric



functions.

山本 修司 (東京大)/Shuji Yamamoto (Tokyo)

On Shintani's ray class invariant for totally real number fields

For a totally real field, we define a ray class invariant  $X(\mathfrak{C})$  from the derivative  $\zeta'(0, \mathfrak{C})$  of the partial zeta function, which was first introduced by Shintani in the case of a real quadratic field. It can be expressed by special values of the multiple sine function, and has a canonical factorization  $X = X_1 \cdots X_n$ , where each factor  $X_i = X_i(\mathfrak{C})$  represents the contribution of a real place. We also show how  $X_i$  behaves when the signature of  $\mathfrak{C}$  at a real place is changed.