

第 12 回広島仙台整数論集会アブストラクト

7月16日(火)

9:30 – 10:30 竹森 翔 (京都大学) / Sho Takemori (Kyoto University)

次数 2, 奇数ウェイトのジークル保型形式の合同式について

A congruence property of odd weight Siegel modular forms of degree 2

Let X_k be a suitable normalization of a Hecke eigen Siegel cusp form of degree two, level 1, weight k . Here X_k is normalized so that the fractional ideal generated by Fourier coefficients of X_k equals to (1) if it is possible. For a half integral semi-positive definite matrix T , $a(T; X_k)$ denotes the T th Fourier coefficient of X_k . Kikuta (Ritsumeikan University), Kodama (Kinki University) and Nagaoka (Kinki University) proved that $(\det T)a(T; X_{35}) \equiv 0 \pmod{23}$ for all T . In this talk, I introduce that there exist higher weight examples. More precisely, I introduce that $(\det T)a(T; X_k) \equiv 0 \pmod{P_k}$ for $k = 47, 71$. Here P_{47} (resp. P_{71}) is a prime above 31 (resp. 47).

10:40 – 11:40 山名 俊介 (九州大学) / Shunsuke Yamana (Kyushu University)

The Siegel-Weil formula for unitary groups over skew fields

The Siegel-Weil formula, which was first proven by Siegel and then generalized by Weil, is an identity between a special value of a certain Eisenstein series and an integral of a theta function under the assumption that the Eisenstein series is absolutely convergent. In this talk I extend the Siegel-Weil formula for unitary groups over skew fields with involution of second kind beyond the range of absolute convergence. If time permits, I will mention applications to the theory of theta correspondence.

11:50 – 12:50 松本 雄也 (東京大学) / Yuya Matsumoto (University of Tokyo)

K3 曲面の良い還元 の判定法について

Good reduction criterion for K3 surfaces

The Néron–Ogg–Shafarevich criterion states that an abelian variety over a CDVF has good reduction if and only if the Galois action on the (l -adic) Tate module satisfies a certain property. We show that a similar criterion holds for good reduction of K3 surfaces (using the l -adic étale cohomology in place of the l -adic Tate module). We also give examples which indicate that the situation is not completely parallel between these (K3 and abelian) cases.

14:00 – 15:00 岡田 健 (京都大学) / Takeshi Okada (Kyoto University)

Abel 多様体のツイストの有限性と Rasmussen-Tamagawa 予想

A finiteness of twists of abelian varieties and Rasmussen-Tamagawa conjecture

Let K be a number field, ℓ a prime number, and let \tilde{K}_ℓ denote the maximal pro- ℓ extension of $K(\mu_\ell)$ unramified outside ℓ . Up to K -isomorphism, there are only finitely many g -dimensional abelian varieties over K whose ℓ -power torsion points are defined over \tilde{K}_ℓ . Rasmussen and Tamagawa that disjoint union of these set of isomorphism classes, varying ℓ , is finite set. They prove the finiteness for some K and g . In this talk, we prove the finiteness of such abelian varieties that is a twist of a fixed abelian variety.

15:15 – 16:15 宮坂 宥憲 (東北大学) / Yuken Miyasaka (Tohoku University)

Honda theory for formal groups of abelian varieties over \mathbb{Q} of GL_2 -type

Honda proved that the formal group associated to the formal completion along the zero section of the Néron model over \mathbb{Z} of an elliptic curve E over \mathbb{Q} is strongly isomorphic over \mathbb{Z} to the formal group obtained from the L -series attached to the l -adic representations of $\text{Gal}(\bar{\mathbb{Q}}/\mathbb{Q})$ on E . Deninger-Nart generalized Honda's theorem to abelian varieties over \mathbb{Q} of GL_2 -type with real multiplication. In this talk, we give a generalization of that with complex multiplication. As an application, we give a method to calculate the coefficients of the L -series attached to the l -adic representations on Jacobian varieties of GL_2 -type.

16:30 – 17:30 吉川 祥 (東京大学) / Shou Yoshikawa (University of Tokyo)

On a relation between the discriminant of an elliptic curve and the torsion points

We give an explicit description of the discriminant of an elliptic curve in terms of the 12-torsion points of the elliptic curve. This description involves the Weil-pairing in a crucial way. As an application, we give a new proof of a Coates' result (under a weaker assumption). This is a joint work with K. Fukuda.

7月17日(水)

9:30 – 10:30 谷田川 友里 (東京大学) / Yuri Yatagawa (University of Tokyo)

On ramification filtration of local fields of equal characteristic

The definition of the ramification filtration of the absolute Galois group G_K of a local field K was generalized by Kato and Abbes-Saito in the case where the residue field F of K is not necessarily perfect, and their methods are different. We consider the case where the characteristic of K is $p > 0$. First, we give a criterion for a character of G_K to annihilate the ramification filtration of Abbes-Saito. Next, we give a new proof of Hasse-Arf theorem using blowing-ups, and finally show the equality between the ramification filtration of Kato and of Abbes-Saito in the perfect F case by using them.

10:40 – 11:40 芳木 武仁 (東京大学) / Takehito Yoshiki (University of Tokyo)

\mathbb{F}_2 係数多項式の素因数の個数の偶奇性を判定するための判別式の一般公式

A general formula for the discriminant of polynomials over \mathbb{F}_2 determining the parity of the number of prime factors

In order to find irreducible polynomials over \mathbb{F}_2 efficiently, the method using Swan's theorem is known. Swan's theorem determines the parity of the number of irreducible factors of a polynomial f over \mathbb{F}_2 with no repeated root, by using the discriminant $D(f) \pmod{8}$, where f is a monic polynomial over \mathbb{Z}_2 such that $\tilde{d}f = f \pmod{2}$. In the lecture, we will give the formula for the discriminant $D(\tilde{d}f) \pmod{8}$ for a polynomial f over \mathbb{F}_2 with no repeated root. By applying this formula to various types of polynomials, we shall get the parity of the number of irreducible factors of them.

11:50 – 12:50 寺門 康裕 (東京大学) / Yasuhiro Terakado (University of Tokyo)

偶数次元射影空間の2重被覆の行列式と分岐因子の判別式

The determinant of a double covering of the projective space of even dimension and the discriminant of the branch locus

The determinant of the Galois action on the étale cohomology of the middle degree of a double covering of the projective space of even dimension defines a quadratic character of

the absolute Galois group of the base field. In this talk, we show that the discriminant of the defining polynomial of the branch locus of the double covering allows us to compute it.

14:00 – 15:00 甲斐 亘 (東京大学) / Wataru Kai (University of Tokyo)

A p -adic exponential map for the Picard group and its application to curves

Let \mathcal{X} be a proper flat scheme over a complete discrete valuation ring \mathcal{O}_k of characteristic $(0, p)$. We define an exponential map from the first cohomology group of $\mathcal{O}_{\mathcal{X}}$ to the Picard group of \mathcal{X} . To be precise, it is an isomorphism between subgroups of each member. This exponential map can be applied to prove a surjectivity property concerning the Albanese variety $Alb_{\mathcal{X}}$ of a smooth variety X over k .

15:15 – 16:15 三原 朋樹 (東京大学) / Tomoki Mihara (University of Tokyo)

p 進バナッハ表現のスペクトル理論と p 進量子論

Spectral theory for p -adic Banach representations and p -adic quantum theory

We give an explicit definition of the functional calculus of a p -adic operator on a p -adic Banach space. The admissibility of the functional calculus yields the notion of the normality of a p -adic operator. The formulation of the normality establishes a p -adic model of Quantum Theory. We study the relation between the normality and the reduction. By the repetition of reductions, we achieve a partial generalisation of Vishik's spectral theory. Note that in the finite dimensional case, the normality corresponds to the diagonalisability of a matrix by a unitary matrix. Therefore our work contains a certain compatibility with the diagonalisability and the reduction. For example, we show that the diagonalisation of the reduction gives a partition of unity corresponding to the reductive spectrum. It decomposes the representation space into the direct sum of subrepresentations. This decomposition is a functorial lift of the eigenspace decomposition of the reduction.

16:30 – 17:30 大川 幸男 (東京大学) / Sachio Ohkawa (University of Tokyo)

正標数の対数的高レベル非可換ホッジ理論について

On logarithmic nonabelian Hodge theory of higher level in characteristic p

Ogus and Vologodsky studied a positive characteristic analogue of Simpson's nonabelian Hodge theory over the complex number field. Now most part of their theory has been generalized to the case of log schemes by Shepler. In this talk, we generalize the global Cartier transform, which is one of the main theorem in nonabelian Hodge theory in positive characteristic, to the case of log schemes and of higher level. This can be regarded as a higher level version of a result of Shepler.

7月18日(木)

9:30 – 10:30 川島 誠 (大阪大学) / Makoto Kawashima (Osaka University)

Evaluation of the dimension of the \mathbb{Q} -vector space spanned by the special values of the Lerch function

For $z \in \mathbb{C}$, $k \in \mathbb{N}$, let $Li_k(z)$ be the k -th polylogarithm function of z . Let s be a positive integer. Nikisin proved that $s + 1$ numbers $1, Li_1(z), \dots, Li_s(z)$ are linearly independent over \mathbb{Q} for some rational numbers z satisfy certain inequalities depending on s . In this talk, we give a generalization of the above result of Nikisin to the case of the Lerch function.

10:40 – 11:40 小野塚 友一 (名古屋大学) / Tomokazu Onozuka (Nagoya University)

Mordell-Tornheim 型 2 重ゼータ関数の 2 乗平均 (立命館大学の岡本卓也氏との共同研究)

Mean value theorems for the Mordell-Tornheim double zeta-function (a joint work with Takuya Okamoto (Ritsumeikan Univ.))

Matsumoto and Tsumura proved mean value theorems for the Euler-Zagier double zeta-function. In this talk, we give mean value theorems for the Mordell-Tornheim double zeta-function.

11:50 – 12:50 三柴 善範 (九州大学) / Yoshinori Mishiba (Kyushu University)

深さ 2 の正標数多重ゼータ値の代数的独立性について

On the algebraic independence of positive characteristic multizeta values of depth 2

Let $\mathbb{F}_q(\theta)$ be the rational function field over the finite field of q elements with variable θ . The positive characteristic multizeta values $\zeta(n_1, \dots, n_d)$ are defined in $\mathbb{F}_q((\theta^{-1}))$ like the classical multiple zeta values in characteristic zero. In positive characteristic, there are several results about the algebraic independence of multizeta values in contrast to the characteristic zero case. In this talk, we study algebraic relations over $\mathbb{F}_q(\theta)$ among the elements $\tilde{\pi}$, $\zeta(n)$ and $\zeta(n, n)$, where $\tilde{\pi}$ is the fundamental period of the Carlitz module. We show that if $q - 1$ does not divide n , then they are algebraically independent over $\mathbb{F}_q(\theta)$ or satisfy some simple relation over $\mathbb{F}_q(\theta)$. We also show that they are algebraically independent over $\mathbb{F}_q(\theta)$ if $q - 1$ does not divide $2n$.

14:00 – 15:00 宮崎 弘安 (東京大学) / Hiroyasu Miyazaki (University of Tokyo)

Special values of zeta functions of singular varieties over finite fields via higher Chow groups

The goal of this talk is to describe special values of zeta functions at $s = 0$ of quasi-projective schemes over finite fields via higher Chow groups.

(i) In the first place, we will recall definitions and some basic properties of zeta function and higher Chow group of schemes over finite fields.

(ii) Next, we will introduce the notion of "weight complex" $W(X)$ of a quasi-projective scheme X over a field k , which was originally defined by Gillet and Soulé. The weight complex allows us to study any quasi-projective scheme X by using Chow motives, which are closely related to projective smooth schemes.

As an example of applications, we will see that if a functor F from the category of smooth projective schemes over k to the category of bounded above complexes of abelian groups satisfies some reasonable conditions, then the homology functor $H_*(F)$ of F can be canonically extended to a functor from the category of quasi-projective schemes (and proper morphisms) over k to the category of abelian groups.

(iii) As a corollary of the above example, we will construct an invariant H_*^W called "weight homology." If Y is a simple normal crossing variety, then the weight homology $H_*^W(Y)$ is isomorphic to the singular homology of the "dual complex" of Y , which is a singular complex naturally associated to Y .

After that, we will construct functorial maps (which we call regulator maps) $\text{Reg}_*(X) : \text{CH}_0(X, *) \rightarrow H_*^W$, where the left hand side of the maps denote the higher Chow groups of zero cycles.

(iv) Finally, we will give an explicit description of the special values of zeta functions at $s = 0$ of any quasi-projective schemes over finite fields by using the regulator maps, and a sketch of proof.

15:15 – 16:15 菅原 弘太郎 (九州大学) / Kotaro Sugahara (Kyushu University)

算術的多様体に対するグローバルアデリックコホモロジー群

Global adelic cohomology groups for arithmetic varieties

Parshin, Beilinson and Huber construct adelic cohomology groups for quasi-coherent sheaves on Noetherian schemes (especially, on algebraic varieties), and show that these correspond to cohomology groups for quasi-coherent sheaves. In this talk, we introduce adelic cohomology groups for quasi-coherent sheaves on arithmetic varieties and give some fundamental properties of those for arithmetic surfaces. It is a joint work with Prof. Lin WENG.

16:30 – 17:30 望月 哲史 (中央大学) / Satoshi Mochizuki (Chuo University)

Simplicial vector fields and dynamical systems

In this lecture, I will explain the idea about simplicial Morse theory and associate simplicial vector fields with Zeta functions. The main topic is how special values of such Zeta functions relate with ranks of Witten-Morse homologies.

7月19日 (金)

9:30 – 10:30 藤澤雄介 (名古屋大学) / Yusuke Fujisawa (Nagoya University)

数体における Apostol のメビウス関数について

On Apostol's Möbius functions in number fields

Let F be a number field, k a positive integer. We consider Apostol's Möbius function of order k in F , which is denoted by $\mu_{k,F}$. We give a formula about the partial sum of $\mu_{k,F}$ and a relation between $|\mu_{k-1,F}|$ and the Ramanujan sum of order k on F . Moreover, we apply our result to counting the number of k -free ideals of the integer ring of F .

10:40 – 11:40 佐野 昂迪 (慶應義塾大学) / Takamichi Sano (Keio University)

Refined abelian Stark conjectures and the equivariant Tamagawa number conjecture

In this talk, we propose a conjecture which generalizes both Gross's and Darmon's conjectural "refined class number formulas". We discuss relations between this conjecture and the equivariant Tamagawa number conjecture. As an application, we give another proof of the "except 2-part" of Darmon's conjecture, which was first proved by Mazur and Rubin.

11:50 – 12:50 野村 次郎 (慶應義塾大学) / Jiro Nomura (Keio University)

Annihilation of Selmer groups for monomial CM-extensions

Let F/k be a finite Galois extension of number fields with Galois group G and A an abelian variety over k . We fix a rational prime p . In this talk, we prove that the Birch and Swinnerton-Dyer conjecture (and some technical assumptions) implies the dual of the classical p -primary Selmer group $\text{Sel}_p(A_F)^\vee$ is annihilated by some suitably defined equivariant L -values if G is isomorphic to the dihedral group of order $4p$. To do that we study the relation between the annihilation of Selmer groups for non-abelian monomial extensions and those for abelian extensions.

14:00 – 15:00 植木 潤 (九州大学) / Jun Ueki (Kyushu University)

絡み目の岩澤不変量と木田の公式

Iwasawa invariants for links and Kida's formula

In a research of the analogy between number theory and low dimensional topology called “arithmetic topology”, Iwasawa theory corresponds to Alexander-Fox theory. In this talk, we translate Kida's formula on Iwasawa λ -invariants of p -extension of cyclotomic \mathbb{Z}_p -fields, into the context of 3-dimensional topology. We first recall some dictionary of the analogy, then give a new proof for the analogue of Iwasawa class number formula, from which we obtain some lemmas, and finally consider an analogue of Kida's formula.

15:15 – 16:15 原 隆 (大阪大学) / Takashi Hara (Osaka University)

3次元多様体に対するカラー-シャーレン理論を巡って

Concerning Culler-Shalen theory for 3-manifolds

Culler-Shalen theory is a classical theory on construction of essential surfaces contained in a topological 3-manifold, which has provided a number of important results in low-dimensional topology. It is of great interest from the arithmetic viewpoint as well since a lot of algebraic methods play remarkable roles there: geometry of character varieties, Bass-Serre theory on trees and so on. After brief review on classical Culler-Shalen theory, we discuss in this talk its extension to higher-dimensional representations of fundamental groups using Bruhat-Tits buildings of higher rank. We then present certain (trials towards) applications to arithmetic topology à la Barry Mazur et Masanori Morishita if time permits. This is a joint work with Takahiro Kitayama (the University of Tokyo).