

## 第17回仙台広島整数論集会アブストラクト

期間：2018年7月10日(火)～7月13日(金)

会場：東北大学大学院理学研究科 川井ホール(数理科学記念館)

7月10日(火)

10:00 – 10:50 加藤 大輝/Hiroki Kato (東京大学/University of Tokyo)

On  $\ell$ -independence of the trace of local monodromy

I will discuss a general solution of  $\ell$ -independence of the alternating sum of the traces of local monodromy. For a variety over a local field, we can consider  $\ell$ -adic Galois representations attached to the variety. Ochiai proved that the alternating sum of the traces of the Weil group action is independent of  $\ell$ . His proof was based on deJong's alteration and the weight spectral sequence. In the case of a curve, Vidal interpreted his weight spectral sequence argument in terms of log geometry to prove  $\ell$ -independence for more general local monodromy actions. I will explain that her argument can be extended to the higher dimensional case by seeing log structures more carefully.

11:00 – 11:50 Ertl Veronika (慶応大学/Keio University)

A vanishing result due to Berthelot, Esnault, and Rülling

In this talk I would like to explain a shorter and more conceptual proof of a result due to Berthelot, Esnault, and Rülling. For a regular, proper, and flat scheme  $X$  over a discrete valuation ring of mixed characteristic  $(0, p)$ , it relates the vanishing of the cohomology of the structure sheaf of the generic fibre of  $X$  with the vanishing of the Witt vector cohomology of its special fibre. We use as a critical ingredient results and constructions by Beilinson and Nekovář–Nizioł related to the h-topos over a  $p$ -adic field.

12:00 – 12:50 山田 一紀/Kazuki Yamada (慶応大学/Keio University)

Comparison between rigid syntomic and crystalline syntomic cohomology for strictly semistable log schemes

We introduce rigid syntomic cohomology for strictly semistable log schemes over the ring of integers of a  $p$ -adic field, and give a comparison theorem between our rigid syntomic cohomology and Nekovář–Nizioł's crystalline syntomic cohomology in the case there exists a good compactification. The main ingredients are modification and generalization of Große-Klönne's rigid Hyodo-Kato theory.

14:00 – 14:50 村上 友哉/Yuya Murakami (東北大学/Tohoku University)

Intersection numbers of modular correspondences

The modular polynomials are the polynomials defined by some products with respect to  $j$ -invariant and whose zero sets are affine planar models of modular curves. Hurwitz shows that the intersection number of two curves defined by modular polynomials can be express as the sum of the class numbers of positive definite binary quadratic forms over  $\mathbb{Z}$ . In this talk, we introduce the modular polynomials for the invariant of elliptic curves with level structures and generalize Hurwitz's result. We also explain that our intersection numbers can be combinatorially written by Fourier coefficients of the Siegel Eisenstein series of degree 2, weight 2 with respect to  $\mathrm{Sp}_2(\mathbb{Z})$ .

15:00 – 15:50 松村 英樹/Hideki Matsumura (慶應大学/Keio University)

A unique pair of triangles

A triangle is called rational if the lengths of its sides are rational numbers. In this talk, we prove that there exists a unique pair of a rational right triangle and a rational isosceles triangle which have the same perimeters and areas (up to similitude). The proof is reduced to determine the set of rational points on a certain hyperelliptic curve. We carry out this task by the method of Chabauty-Coleman and the 2-descent on the Jacobian variety of a hyperelliptic curve with a help of MAGMA. Here, the method of Chabauty-Coleman gives an upper bound of the number of rational points on a hyperelliptic curve if its Mordell-Weil rank is "sufficiently small", which is verified for the above hyperelliptic curve by the 2-descent. This is a joint work with Yoshinosuke Hirakawa.

16:10 – 17:00 東山 和巳/Kazumi Higashiyama (京都大学/Kyoto University)

The absolute anabelian geometry of pro- $p$  fundamental groups of second configuration spaces

The  $n$ -th configuration space of a hyperbolic curve is the scheme which parametrizes  $n$ -tuples of pairwise distinct points in the hyperbolic curve. Mochizuki proved the Grothendieck conjecture for hyperbolic curves. We discuss a certain pro- $p$  version of the Grothendieck conjecture for hyperbolic curves. In this talk, we reconstruct group-theoretically the function field of a hyperbolic curve of type  $(0, 3)$  from the pro- $p$  fundamental group of the associated second configuration space equipped with the collection of decomposition groups.

17:10 – 18:00 室谷 岳寛/Takahiro Murotani (京都大学/Kyoto University)

絶対版 Grothendieck 予想への  $p$  進解析的アプローチ/A  $p$ -adic analytic approach to the absolute Grothendieck conjecture

Let  $K$  be a field,  $G_K$  the absolute Galois group of  $K$ ,  $X$  a hyperbolic curve over  $K$ , and  $\pi_1(X)$  the étale fundamental group of  $X$ . In absolute anabelian geometry, we consider the following problem (the absolute Grothendieck conjecture): Is it possible to recover  $X$  group-theoretically, solely from  $\pi_1(X)$  (not  $\pi_1(X) \rightarrow G_K$ )? When  $K$  is a  $p$ -adic field (i.e. a finite extension of  $\mathbb{Q}_p$ ), this problem is open. To consider this problem, we introduce a certain  $p$ -adic analytic invariant defined by Serre (which we call  $i$ -invariant). Then, the absolute  $p$ -adic Grothendieck conjecture can be reduced to the following problems: (A) determining whether a proper hyperbolic curve admits a rational point from the data of  $i$ -invariants of the sets of rational points of the curve and its coverings; (B) recovering the  $i$ -invariant of the set of rational points of a proper hyperbolic curve group-theoretically. Our main results give a complete affirmative answer to (A) and a partial affirmative answer to (B). In this talk, we present these results and sketches of their proofs.

7月11日(水)

9:30 – 10:20 河野 孝彦/Kohno Takahiko (立教大学/Rikkyo University)

ガウス和の乗法的関係式について/On the multiplicative relations of Gauss sums

Let  $m$  be a positive integer and  $p$  a prime number with  $p \equiv 1 \pmod{m}$ . Let  $K_m$  be the  $m$ -th cyclotomic field and fix a prime ideal  $P$  of  $K_m$  dividing  $p$ . Let  $\chi_m : (\mathcal{O}_{K_m}/P)^\times \rightarrow \mu_m$  be the  $m$ -th power residue symbol. Then for any  $a \in \mathbf{Z}/m\mathbf{Z} - \{0\}$  the Gauss sum  $\tau_m(a)$  is defined to be the sum  $\sum_{k=1}^{p-1} \chi_m(k)^a \exp(2\pi i k/p)$ . Although it is usually hard to determine the Gauss sums explicitly, there exist some sequences of integers  $\alpha = (c_1, c_2, \dots, c_{m-1})$  such that  $\tau_m(\alpha) := \prod_{a=1}^{m-1} \tau_m(a)^{c_a} = \zeta p^s$ , where  $\zeta$  is a root of unity in  $K_m$  and  $s = (c_1 + \dots + c_{m-1})/2 \in \mathbf{Z}$ . In this case, one can easily evaluate  $\zeta^2$  using the Hasse-Davenport relations. However, it is a number theoretically subtle problem to determine which square root of  $\zeta^2$  appears in  $\tau_m(\alpha)$ . Indeed, a theorem of Deligne on the products of some special values of  $\Gamma$ -functions shows that this problem is closely related to the decomposition of the prime ideal  $P$  in a quadratic extension  $L_m$  of  $K_m$ . In this talk, I will study  $L_m$  when  $m$  is a product of two distinct primes.

10:30 – 11:20 伊東 邦大/Kunihiro Ito (東北大学/Tohoku University)

Landen 型接続公式の拡張と multi-indexed poly-Bernoulli 数の関係式/A generalization of the Landen connection formula and relations among multi-indexed poly-Bernoulli numbers

The Landen connection formula for multiple polylogarithms, which is shown by Okuda-Ueno, gives much information on multiple zeta functions and its special values. In this talk, we present a generalization of the Landen connection formula for multiple polylogarithms of shuffle type in several variables with certain classes of indices. We also mention its application to relations among multi-indexed poly-Bernoulli numbers.

11:30 – 12:20 原田 遼太郎/Ryotaro Harada (名古屋大学/Nagoya University)

Bernoulli-Carlitz 数の一般化について/On generalizations of Bernoulli-Carlitz numbers

The Bernoulli-Carlitz numbers were introduced by L. Carlitz in 1935 as function field analogues of Bernoulli numbers. In this talk, we introduce multi-poly-Bernoulli-Carlitz numbers which are generalizations of Bernoulli-Carlitz numbers and function field analogues of multi-poly-Bernoulli numbers of Imatomi-Kaneko-Takeda. I also explicitly describe multi-poly-Bernoulli Carlitz numbers

in terms of the Carlitz factorial and the Stirling-Carlitz numbers of the second kind and show the vanishing condition of them. Furthermore, I explain their relationships with function field analogues of finite multiple zeta values.

13:50 – 14:40 平川 義之輔/Yoshinosuke Hirakawa (慶應大学/Keio University)

A heuristic recipe for insoluble equations of the form  $x^3 + py^3 = Cz^p$

The contents of this talk is based on the joint work with Yosuke Shimizu. Since the encounter with the Pythagorean equation or the Fermat equations, a considerable number of researches have been done on the Diophantine equations of the form  $Ax^l + By^m = Cz^n$ . For instance, in their seminal paper published in 1995, Darmon and Granville proved that whenever  $1/l + 1/m + 1/n < 1$ , the equation  $Ax^l + By^m = Cz^n$  has only finitely many coprime integer solutions  $(x, y, z)$  for each triple of non-zero integers  $(A, B, C)$ . Let  $p > 3$  be a prime number. In this talk, we introduce a heuristic recipe for an integer  $C$  for which  $x^3 + py^3 = Cz^p$  has no coprime integer solutions (more strongly, it violates a certain Hasse principle). Key ingredients are a certain  $p$ -adic property of the fundamental unit of  $\mathbb{Q}(p^{1/3})$  and the decomposition law of prime numbers in  $\mathbb{Q}(p^{1/3})$ . Along with examples, we would like to mention some interesting phenomena which the speaker encountered in construction of examples.

14:50 – 15:40 石本 和基/Kazuki Ishimoto (神戸大学/Kobe University)

Fourier transform for prehomogeneous vector spaces over a finite field: cubic cases

Let  $V$  be a finite dimensional representation of a reductive algebraic group  $G$  defined over a field  $K$ . When there exists a  $G(\overline{K})$ -orbit of  $V(\overline{K})$  which is Zariski open, we refer to the pair  $(G, V)$  as a prehomogeneous vector space. Here  $\overline{K}$  is the algebraic closure of  $K$ . Let  $\phi$  be a  $\mathbb{C}$ -valued function on  $V(\mathbb{F}_q)$  which is  $G(\mathbb{F}_q)$ -invariant. For  $\phi$  we can define the Fourier transform  $\widehat{\phi}$ . In this talk, we give explicit formulas of the Fourier transform for several “cubic cases” prehomogeneous vector spaces and explain an application of Fourier transform.

16:00 – 16:50 井上 翔太/Shota Inoue (名古屋大学/Nagoya University)

Some explicit formulas for partial sums of Möbius functions

The purpose of this talk is to give some explicit formulas involving Möbius functions, which may be known under the generalized Riemann Hypothesis, but unconditional in this talk. Concretely, we discuss explicit formulas of partial sums of the Möbius function in arithmetic progressions and partial sums of the Möbius functions on an Abelian number field  $K$ . In addition, we also discuss certain finite Euler products to obtain these explicit formulas.

17:00 – 17:50 角濱 寛隆/Hiroataka Kakuhama (京都大学/Kyoto University)

On the local factors of irreducible representations of Quaternionic unitary groups

Let  $G$  be a unitary group of a hermitian or skew-hermitian space over a quaternion algebra of a local field  $F$  of characteristic 0. We consider local factors (the  $L$ -factor, the  $\epsilon$ -factor and the  $\gamma$ -factor) of an irreducible representation of  $(G \times \mathrm{GL}_1)(F)$ . The motivation is to define them with analytic tools. From the analog of classical theory and the aspect of the local Langlands correspondence, they are expected to satisfy some natural properties. In this talk, I will prove that some expected properties determines the  $\gamma$ -factor uniquely, and construct the  $\gamma$ -factor satisfying them. Then, I will explain that the  $\gamma$ -factor determines the  $L$  and  $\epsilon$ -factor. This extends a work of Lapid and Rallis for general linear groups, orthogonal groups, symplectic groups, and unitary groups.

7月12日(木)

9:30 – 10:20 加藤 裕基/Yuki Kato (宇部高専/Ube College)

Oriented completion and algebraic cobordism

The complex cobordism  $\mathbf{MU}$  is the universal complex oriented cohomology theory. In the framework of stable  $\mathbb{A}^1$ -homotopy theory, Voevodsky defined the algebraic cobordism  $\mathbf{MGL}$  as the algebraic analogy of the spectrum of the complex cobordism  $\mathbf{MU}$ . Panin, Pimenov and Röndigs proved that  $\mathbf{MGL}$  is the universal oriented cohomology theory. In this talk, we study and reformulate the algebraic cobordism by using categorical language. Actually, we construct the *oriented completion functor* and obtain the algebraic cobordism as oriented completion of the motivic sphere spectrum.

10:30 – 11:20 坂垣内 誠/Makoto Sakagaito (IISER Mohali)

On a generalized Brauer group in mixed characteristic cases

We define a generalization of the Brauer group  $H_{\mathbb{B}}^n(X)$  for an equi-dimensional scheme  $X$  and  $n > 0$ . In the case where  $X$  is the spectrum of a local ring of a smooth algebra over a discrete valuation ring,  $H_{\mathbb{B}}^n(X)$  agrees with the étale motivic cohomology  $H_{\text{ét}}^{n+1}(X, \mathbb{Z}(n-1))$ . We prove a weak form of the Gersten-type conjecture for the generalized Brauer group for a local ring of a smooth algebra over a mixed characteristic discrete valuation ring and introduce its application.

11:30 – 12:20 岡野 恵司/Keiji Okano (都留文科大学/Tsuru University)

虚二次体上の  $\mathbb{Z}_p^2$ -拡大の不分岐岩澤加群について/On the unramified Iwasawa modules of the  $\mathbb{Z}_p^2$ -extensions over imaginary quadratic fields

Let  $p$  be an odd prime number. The unramified Iwasawa module (i.e., the Galois group of the maximal unramified abelian extension)  $X$  of any algebraic number field  $k$  is conjectured as to be pseudo-null, which is known as “Generalized Greenberg’s Conjecture”. Even if this conjecture is true, we still know little about properties of  $X$ . Our aim is to get some properties of  $X$ . In this talk, we describe the order of a Galois coinvariant of  $X$  when  $k$  is an imaginary quadratic field in which  $p$  splits completely. Also, as its application, we consider some characteristic ideals of restricted Iwasawa modules. Note that these results need not assume Generalized Greenberg’s Conjecture is true. This is joint work with Takashi Miura, Kazuaki Murakami and Rei Otsuki.

13:50 – 14:40 富安 亮子/Ryoko Oishi-Tomiyas (山形大学/Yamagata University)

同じ  $\mathbb{Z}$  上表現を持つ正定値 3 変数 2 次形式に関する Kaplansky 予想について/On a Kaplansky conjecture concerning positive-definite ternary quadratic forms with the same representations over  $\mathbb{Z}$ .

Kaplansky conjectured that if two positive-definite real ternary quadratic forms have completely identical representations over  $\mathbb{Z}$ , they are constant multiples of regular forms, or is included in either of two families parametrized by  $\mathbb{R}$  (so called, hexagonal and rhombohedral families). The two families have been also known in crystallography, where algorithms for low-rank lattices are daily used for data analysis. Our results consist of computational and theoretical parts. Firstly, the result of an exhaustive search for such quadratic forms is presented, in order to provide a concrete version of the Kaplansky conjecture. Secondly, we prove that if two pairs of ternary quadratic forms have the identical simultaneous representations over  $\mathbb{Q}$ , their constant multiples are equivalent over  $\mathbb{Q}$ . This was motivated by the question why the other families were not detected in the search. In the proof, the parametrization of quartic rings and their resolvent rings by Bhargava is used to discuss pairs of ternary quadratic forms.

14:50 – 15:40 中屋 智瑛/Tomoaki Nakaya (九州大学/Kyushu University)

The number of linear factors of supersingular polynomials

The set of prime numbers  $p$  such that the supersingular  $j$ -invariants in characteristic  $p$  are all contained in the prime field is finite. And it is well known that this set of primes coincides with the set of prime divisors of the order of the Monster simple group. In this talk, I will present analogous coincidence of supersingular invariants in level 2 and 3 and the orders of the Baby monster group and the Fischer group. The proof uses a connection between the number of supersingular invariants and class numbers of imaginary quadratic fields.

16:00 – 16:50 澤田 晃一郎/Koichiro Sawada (京都大学/Kyoto University)

与えられた基本群を持つ多重双曲的曲線の同型類の有限性/Finiteness of isomorphism classes of hyperbolic polycurves with prescribed fundamental groups

A hyperbolic polycurve is a successive extension of families of hyperbolic curves. Hoshi proved the Grothendieck conjecture (in anabelian geometry) for hyperbolic polycurves of dimension  $\leq 4$ , i.e., roughly speaking, a hyperbolic polycurve of dimension  $\leq 4$  over a certain type of a field is completely determined by its arithmetic fundamental group. However, the Grothendieck conjecture for hyperbolic polycurves of dimension  $\geq 5$  is still open. In this talk, we show that the isomorphism class of a hyperbolic polycurve (of any dimension) over a certain type of a field is determined by its fundamental group up to finitely many possibilities.

17:00 – 17:50 長町 一平/Ippeï Nagamachi (東京大学/University of Tokyo)

Criteria for good reduction of hyperbolic polycurves

We give good reduction criteria for hyperbolic polycurves, i.e., successive extensions of families of curves, under mild assumption. These criteria are higher dimensional versions of the good reduction criterion for hyperbolic curves given by Oda and Tamagawa. In this talk, we construct homotopy exact sequences by using intermediate quotient groups of geometric étale fundamental groups of hyperbolic polycurves.

7月13日(金)

9:30 – 10:20 太田 和惟/Kazuto Ota (慶應大学/Keio University)

Anticyclotomic Iwasawa main conjecture for modular forms

We study the Iwasawa main conjecture which relates a Selmer group with the anticyclotomic  $p$ -adic  $L$ -function constructed by Bertolini et al. The goal of this talk is to describe a joint work with Shinichi Kobayashi, which shows a half part of the conjecture.

10:30 – 11:20 岩佐 亮明/Ryomei Iwasa (東京大学/University of Tokyo)

Rational isomorphisms between relative  $K_0$ -groups and Chow groups with modulus.

This is a joint work with Wataru Kai. Let  $X$  be an affine algebraic smooth scheme and  $D$  an effective Cartier divisor on  $X$ . I'll explain a construction of a cycle class map from Binda-Saito's Chow group with modulus  $CH^*(X|D)$  to a suitable subquotient of the relative  $K_0$ -group  $K_0(X, D)$ . I show that the cycle class map is surjective and the kernel is torsion, which generalizes Grothendieck's theorem to relative setting.

11:30 – 12:20 小原 まり子/Mariko Ohara (信州大学/Shinshu University)

On graded  $\mathbb{E}_\infty$ -rings and projective schemes in spectral algebraic geometry

In this talk, I define the notion of grading on  $\mathbb{E}_\infty$ -rings, and explain the first properties such as localization with one element. By using this grading, I will define projective schemes in spectral algebraic geometry. In special cases, it is equivalent to projective spaces (schemes) in the sense of Lurie's. As an application, I define some "finiteness" conditions and show a generalization of the Serre theorem which provides an equivalence between  $\infty$ -category of the finitely presented module sheaves over a projective scheme associated to a graded  $\mathbb{E}_\infty$ -ring and a certain localized  $\infty$ -category of graded modules over the graded  $\mathbb{E}_\infty$ -ring.

13:30 – 14:20 関 真一郎/Shin-ichiro Seki (東北大学/Tohoku University)

A new proof of the duality of usual/finite multiple zeta values using "connected sums"

The duality of multiple zeta values has been proved by using iterated integral representations. In this talk, we present a new proof of the duality without using iterated integrals. The proof is done by introducing certain "connected sums". This proof is also applicable to dualities for  $q$ -multiple zeta values or finite multiple zeta values and to their generalizations of Ohno-type. As a byproduct, we give sum formulas for modulo  $p^2$  version of finite multiple zeta values. This is a joint work with Shuji Yamamoto.

14:30 – 15:20 小野 雅隆/Masataka Ono (慶應大学/Keio University)

Multiple zeta functions associated with 2-colored rooted trees

In our recent work, we introduced a combinatorial object and finite sum associated with them which we call finite multiple zeta values associated with 2-colored rooted trees and gave a unified interpretation to some types of finite multiple zeta values. In this talk, we introduce multiple zeta function associated with 2-colored rooted tree and discuss its analytic properties. In particular, we show that they can be continued meromorphically to the whole space and we can give the set of possible singularities of them in terms of combinatorial information of 2-colored rooted trees in some special cases.

15:40 – 16:30 跡部 発/Hiraku Atobe (東京大学/University of Tokyo)

局所宮脇リフトについて/On the local Miyawaki liftings

In 2006, Ikeda constructed a certain lifting of Siegel modular forms to approach a conjecture predicted by Miyawaki in 1992. This lifting is now called Miyawaki lifting. In this talk, we define a local analogue of Miyawaki liftings, and explain its properties.

16:40 – 17:30 大井 雅雄/Masao Oi (東京大学/University of Tokyo)

剰余標数の大きな古典群の局所 Langlands 対応の深度保存則/Depth preserving property of the local Langlands correspondence for classical groups in a large residual characteristic

Let  $G$  be a classical group over a  $p$ -adic field  $F$ . Then the local Langlands correspondence for  $G$  gives a natural correspondence between irreducible smooth representations of  $G(F)$  and  $L$ -parameters of  $G$  (which are variants of Galois representations of  $F$ ). It is believed that the local Langlands correspondence satisfies a lot of natural properties beyond its characterization. One of such phenomena is the depth preserving property. We can define the notion of depth, which is a numerical invariant, for both of irreducible representations and  $L$ -parameters. Then it is expected that the local Langlands correspondence for classical groups preserves these invariants at least in a large residual characteristic. In this talk, I will give a partial answer to this problem, and also give a complete answer in the case of unitary groups.