Prediction and Estimation of Random Fields on Quarter Planes

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We present solutions of several prediction problems for random fields (2-D processes) which extend some nonstandard prediction problems for a stationary time series (1-D process) based on the modified pasts. The solutions lead to informative and explicit expressions involving the AR and MA parameters (Nakazi, 1984; Pourahmadi, Inoue and Kasahara, 2006) as summarized in the following:

Theorem: Let $\{X(t), t \in \mathbb{Z}\}$ be a nondeterministic stationary process with the innovation process $\{\varepsilon(t), t \in \mathbb{Z}\}$, innovation variance σ^2, MA and AR parameters $\{b_k\}$ and $\{a_k\}$, respectively. Then, for $h > 0$ an integer the prediction error variance of $X(0)$ based on

(a) $I_1 = \{X(t), t \leq -1, t = h, t \neq 0\}$, having only one additional observation at time h, is

$$\mathrm{Var}\left\{X(0) - \hat{X}_{I_1}(0)\right\} = \sigma^2 \frac{1 + b_1^2 + b_2^2 + \cdots + b_{h-1}^2}{1 + b_1^2 + b_2^2 + \cdots + b_{h-1}^2 + b_h^2}.$$

(b) $I_2 = \{X(t), t \leq h, t \neq 0\}$, having h additional observations at times $1, \cdots, h$, is

$$\mathrm{Var}\left\{X(0) - \hat{X}_{I_2}(0)\right\} = \sigma^2 \frac{1}{1 + a_1^2 + a_2^2 + \cdots + a_h^2}.$$

(c) $I_3 = \{X(t), t \leq -1, t \neq -h, t \neq 0\}$, missing only one observation at time $-h$, is

$$\mathrm{Var}\left\{X(0) - \hat{X}_{I_3}(0)\right\} = \sigma^2 \frac{1 + a_1^2 + a_2^2 + \cdots + a_{h-1}^2 + a_h^2}{1 + a_1^2 + a_2^2 + \cdots + a_{h-1}^2}.$$

These expressions involving $\sum_{i=0}^{h} b_i^2$ and $\sum_{i=0}^{h} a_i^2$ are reminiscent of the h-step ahead prediction error variance $\sigma^2 \sum_{i=0}^{h} b_i^2$, and reveal the effect (worth) of observations in prediction. Using the Wold decomposition of stationary random fields, their multi-step ahead prediction errors and variances, solutions are provided for various nonstandard prediction problems when a number of observations are either added to or deleted from the quarter-plane past. Unlike the time series situation, the prediction error variances for random fields seems to be expressible only in terms of the MA parameters, attempts to express them in terms of the AR parameters runs into a mysterious projection operator which captures the nature of the "edge-effects" encountered in estimation of random fields. These prediction problems provide useful information for assessing worth of observations in the spatial setting, and are closely related to the design issue or network site selection in the environmental, geostatistical and engineering applications (Zimmerman, 2006). For a given data, a prediction methodology is implemented by fitting exponential models (Bloomfield, 1973; Solo, 1986) to the spectrum and then using recursive formulas expressing the AR, MA and predictor coefficients in terms of the cepstral coefficients of the random field. The procedure is illustrated using a simulation study and application to real data.