AUTOMORPHISM GROUPS OF CERTAIN ENRIQUES SURFACES: COMPUTATIONAL DATA

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1. Introduction

This note is the explanation of the computation data that are used to obtain the main results of the paper

[BS] S. Brandhorst, I. Shimada: Automorphism groups of certain Enriques surfaces.

The data is available from the author's webpage [2] as plain text files. The data consists of the following items.

`GramL10`, `oneBP`, `irecs`, `Enrs`, `E6EnrRDPs` .

They are written in 3 files:

`Preliminaries.txt`	:	`GramL10`, `oneBP`,
`irecs.txt`	:	`irecs`,
`Enrs.txt`	:	`Enrs`,
`E6EnrRDPs.txt`	:	`E6EnrRDPs`.

The data are made by `GAP` (see [3]). In particular, the `Record` format of `GAP` is used everywhere. The results of Algorithms in Sections 6.3 and 6.4 of [BS] are too large to be put on a webpage, and hence we omit them.

2. The file `Preliminaries.txt`

In the file `Preliminaries.txt`, we have the items `oneBP` and `oneBP`.

The item `GramL10` is the Gram matrix of the even unimodular hyperbolic lattice L_{10} of rank 10 with respect to the basis $\{e_1, \dots, e_{10}\}$ given in Figure 1.1 of the paper [BS]. Throughout this note, every computation data about the lattice L_{10} is expressed in terms the basis $\{e_1, \dots, e_{10}\}$.

The item `oneBP` is the number

$$\texttt{oneBP} := 2^{21} \cdot 3^5 \cdot 5^2 \cdot 7 \cdot 17 \cdot 31 = 46998591897600,$$

which is the unit of volume of chambers in $\mathcal{P}_{10}$.

3. The file `irecs.txt`

The list `irecs` describes the primitive embeddings

$$\iota\colon L_{10}(2) \hookrightarrow L_{26}$$

classified in [1], except for the primitive embedding of type `infty`. Thus `irecs` consists of 16 records `irec`, and each has the following items. We use the notions and notation in [1] and [BS]. Recall that R_ι denote the orthogonal complement of the image of ι in L_{26}.

- `irec.name` is the name of ι, which is one of the strings `"12A"`, `"12B"`, ..., `"96C"`.
- `irec.GramL26` is the Gram matrix of L_{26} with respect to a certain fixed basis. We use this basis for other data in this record `irec`.
- `irec.embS` is the 10×26 integer matrix M such that $v \mapsto vM$ gives the embedding $\iota\colon L_{10}(2) \hookrightarrow L_{26}$, where L_{10} is equipped with the basis $\{e_1, \dots, e_{10}\}$.
- `irec.embR` is the 16×26 integer matrix M' such that $v \mapsto vM'$ gives the embedding of R_ι into L_{26} with respect to a certain fixed basis of R_ι.
- `irec.GramR` is the Gram matrix of R_ι with respect to the fixed basis.
- `irec.rootsR` is the list of roots of R_ι.
- `irec.rootstypeR` is the ADE-type of the system of roots of R_ι.
- `irec.m4RVects` is the list of (-4)-vectors of R_ι.
- `irec.weyl` is the Weyl vector $\mathbf{w} \in L_{26}$ such that $D_0 := \iota_{\mathcal{P}}^{-1}(C(\mathbf{w}))$ is an induced chamber in $\mathcal{P}(L_{10})$, where $C(\mathbf{w}) \subset \mathcal{P}(L_{26})$ is the Conway chamber corresponding to $\mathbf{w}$.
- `irec.weylprime` is another Weyl vector $\mathbf{w}' \in L_{26}$ such that $\langle \mathbf{w}, \mathbf{w}' \rangle_{26} = 1$ and that $a_{26} = 2\mathbf{w} + \mathbf{w}'$ is an interior point of $C(\mathbf{w})$.
- `irec.walls` is the list of roots of L_{10} defining the walls of the induced chamber D_0.
- `irec.alphas` is the list of pairs $\alpha = (r, v)$, where r is a root of L_{10} defining a wall of D_0 and v is a (-4)-vector of R_ι such that $(r+v)/2 \in L_{26}$.
- `irec.volindex` is $1_{\mathrm{BP}}/\mathrm{vol}(D_0)$, where $1_{\mathrm{BP}} = 46998591897600$.
- `irec.OL10D0` is the list of matrices of the elements of $\mathrm{O}(L_{10}, D_0)$.
- `irec.ample` is a primitive vector of L_{10} belonging to the interior of D_0 that is fixed under the action of $\mathrm{O}(L_{10}, D_0)$.
- `irec.codim2facewallpairs` is the list of pairs of two indexes $\{i, j\}$ such that $D_0 \cap (r_i)^\perp \cap (r_j)^\perp$ is a face of codimension 2 of D_0, where r_i and r_j are the ith and the jth elements of `irec.walls`, respectively.
- `irec.isotropicrays` is the list of primitive isotropic rays $f \in L_{10}$ contained if $\overline{D}_0$.
- `irec.wallrecs` is the list of records `wrec`, each of which contains the data about a wall w of D_0. Let `wrec` be a record in `irec.wallrecs` corresponding to a wall $w = D_0 \cap (r)^\perp$ of D_0. Then `wrec` has the following items. See the proof of Proposition 2.7 of [1] for notation.
  - `wrec.r` is the root r of L_{10} that defines the wall $w = D_0 \cap (r)^\perp$.
  - `wrec.rlifts` is the list of roots $\tilde{r}$ of L_{26} such that $\langle \mathbf{w}, \tilde{r} \rangle_{26} = 1$ and $\iota_{\mathcal{P}}^{-1}((\tilde{r})^\perp) = (r)^\perp$.
  - `wrec.thegtilde` is the isometry $\tilde{g} \in \mathrm{O}^{\mathcal{P}}(L_{26})$ such that $\tilde{g}$ preserves the image of $\iota\colon L_{10}(2) \hookrightarrow L_{26}$ and that its restriction $\tilde{g}|L_{10}(2)$ to $L_{10}(2)$ maps D_0 to the induced chamber adjacent to D_0 across the wall $w = D_0 \cap (r)^\perp$. The existence of this isometry proves Proposition 2.7 of [1].
  - `wrec.theg` is the isometry $g = \tilde{g}|L_{10}(2)$ of L_{10}. We can check that this isometry is equal to the reflection with respect to the root `wrec.r`.
- `irec.facerecs` is a complete list of representatives of orbits of the action of $\mathrm{O}(L_{10}, D_0)$ on the set of faces of D_0. Here we include isotropic rays

contained in $\overline{D}_0$ to the set of faces of D_0. The list `irec.facerecs` consists of records `frec`, each of which expresses a representative face f of an orbit of faces under the action of $\mathrm{O}(L_{10}, D_0)$. The record `frec` has the following items:

- `frec.dim` is the dimension of f.
- `frec.walls` is the list of indexes i such that $f \subset (r_i)^\perp$, where $D_0 \cap (r_i)^\perp$ is the ith wall in the list `irec.walls` above.
- `frec.adetype` is the ADE-type of the Dynkin diagram formed by the roots r_i that define walls of D_0 containing f. This ADE-type is an ordinary ADE-type when f is not an isotropic ray, whereas it is an *affine* ADE-type when f is an isotropic ray.
- `frec.basis` is a basis of the minimal linear space $\langle f \rangle$ of $L_{10} \otimes \mathbb{R}$ containing f. When $\dim f = 1$ so that `frec.basis` consists of a single element v, we can determine whether f is an isotropic ray or not by calculating $\langle v, v \rangle$.
- `frec.orbitsize` is the size of the orbit of f under the action of $\mathrm{O}(L_{10}, D_0)$.

4. Generalities

Let $\pi\colon X \to Y$ be the universal covering of a complex Enriques surface Y, and ε the deck-transformation of π. The lattice S_X is equipped with a basis.

4.1. ADE**-types.** An ADE-type is a list of strings `"A1"`, `"A2"`, ..., `"E8"`.

4.2. **Elements of** $\mathrm{aut}(Y)$**.** An element g of $\mathrm{aut}(Y)$ is expressed by a record `grec` that has items `grec.gX` and `grec.gY`. Let $\tilde{g} \in \mathrm{aut}(X, \varepsilon)$ be an element of $\mathrm{aut}(X)$ commuting with ε such that $\tilde{g}|S_Y = g$ and that $\tilde{g}$ acts on the discriminant group $S_X^\vee / S_X$ trivially. The item `grec.gX` is the matrix representing the action of $\tilde{g}$ on S_X, and the item `grec.gY` is the matrix representing the action of g on $S_Y \cong L_{10}$. Let `emb` be the matrix such that $v \mapsto v \cdot$`emb` is the embedding $\pi^*\colon S_Y(2) \cong L_{10}(2) \hookrightarrow S_X$. Then we have `emb` $\cdot$ `gX` $=$ `gY` $\cdot$ `emb`.

4.3. **Smooth rational curves on** Y**.** A smooth rational curve C on Y is expressed by a record `rrec` that has items `rrec.ratY` and `rrec.lifts`. The item `rrec.ratY` is the class $[C] \in S_Y$ of the curve C, and the item `rrec.lifts` is the pair of classes $[\widetilde{C}_1], [\widetilde{C}_2] \in S_X$ of the two irreducible components of $\pi^{-1}(C) = \widetilde{C}_1 + \widetilde{C}_2$. Let `emb` be as above. Then we have `ratY` $\cdot$ `emb` $=$ `lifts`$[1]$ $+$ `lifts`$[2]$.

4.4. **The lists** V_0 **and** $\mathcal{H}$**.** The outputs V_0 and $\mathcal{H}$ of Procedure 4.1 of the paper [BS] are described by the following data. In our applications, a vertex of the graph (V, E) is an $L_{26}/S_Y(2)$-chamber. For each Enriques surface Y, we fix an $L_{26}/S_Y(2)$-chamber D_0 contained in Nef_Y, and hence each $L_{26}/S_Y(2)$-chamber D is expressed as D_0^τ by some isometry $\tau \in \mathrm{O}^{\mathcal{P}}(L_{10})$. (The isometry τ is unique up to left multiplications of elements of $\mathrm{O}(L_{10}, D_0)$.) The group G acting on (V, E) is a subgroup of $\mathrm{aut}(Y)$, and hence each element of G is expressed by a record `grec` in the way explained in Section 4.2.

The ordered list V_0 of vertices is expressed by an ordered list `V0` of records `chamrec`. Each `chamrec` has the following items. Let D be the $L_{26}/S_Y(2)$-chamber expressed by `chamrec`.

- `chamrec.pos` is the position of the `chamrec` in `V0`. **Caution:** The position starts from 1 so that `chamrec` is `V0[chamrec.pos]`, whereas, in the explanation of Procedure 4.1 in the paper [BS], the position starts from 0.
- `chamrec.from` and `chamrec.adjpos` indicate from which chamber the chamber D is obtained. Namely, the chamber D is the chamber adjacent to the chamber D_{prev} expressed by a record `prevchamrec` in `V0` whose position `prevchamrec.pos` is equal to `chamrec.from`, and the wall across which D is adjacent to D_{prev} is the `chamrec.adjpos`-th wall in the list `prevchamrec.adjrecs` of adjacent chambers of D_{prev} (see below). If the chamber D is the initial chamber of the list V_0 (that is, `chamrec.pos` is 1), both of `chamrec.from` and `chamrec.adjpos` are `"none"`.
- `chamrec.taug` is a matrix $\tau \in \mathrm{O}^{\mathcal{P}}(L_{10})$ such that $D = D_0^{\tau}$.
- `chamrec.autcham` is the list of elements of the stabilizer subgroup $T_G(D, D)$ of D in the group G. Since G is a subgroup of $\mathrm{aut}(Y)$, each element of `chamrec.autcham` is expressed by a record `grec`.
- `chamrec.adjrecs` is the list of adjacent chambers D' of D. Each element of `chamrec.adjrecs` is a record `adjrec` with the following items.
  - `adjrec.wall` is the root r' of S_Y defining the wall of D across which D' is adjacent to D. When we apply our algorithm to the calculation of the group $\mathrm{aut}(Y)$ (Section 6.1 of [BS]) or the stabilizer subgroup of an elliptic fibration $\phi\colon Y \to \mathbb{P}^1$ (Section 6.4 of [BS]), the chamber D' is the image $D^{s_{r'}}$ of D by the reflection $s_{r'}$ with respect to the root r'. When we apply our algorithm to the calculation of the stabilizer subgroup of a smooth rational curve C on Y (Section 6.3 of [BS]), the wall is in fact the face $D \cap (r)^{\perp} \cap (r')^{\perp}$ of D with codimension 2, where $r = [C]$, and the chamber D' is
  $$\begin{cases} D^{s_{r'}} & \text{if } \langle r, r' \rangle = 0, \\ D^{s_r s_{r'}} & \text{if } \langle r, r' \rangle = 1, \end{cases}$$
  where s_r is the reflection with respect to the root $r = [C]$. When we apply the algorithm to the graph (V_Γ, E_Γ) and the group G_Γ associated with an RDP-configuration Γ (Section 7.1 of [BS]), the root r' is a defining root of a wall of D such that
  $$w_f := \mathcal{P}_{\langle \Gamma \rangle^{\perp}} \cap D \cap (r')^{\perp}$$
  is the wall of the $L_{26}/\langle \Gamma \rangle^{\perp}(2)$-chamber $f := \mathcal{P}_{\langle \Gamma \rangle^{\perp}} \cap D$. Note that f is a face of D with dimension $10 - |\Gamma| = \dim \mathcal{P}_{\langle \Gamma \rangle^{\perp}}$ (that is, f contains a non-empty open subset of $\mathcal{P}_{\langle \Gamma \rangle^{\perp}}$), whereas w_f is a face of D with dimension $9 - |\Gamma|$ contained in f. The adjacent chamber is the unique $L_{26}/S_Y(2)$-chamber D' such that $D' \neq D$, that D' is contained in Nef_Y, that D' contains w_f, and that $\mathcal{P}_{\langle \Gamma \rangle^{\perp}} \cap D'$ contains a non-empty open subset of $\mathcal{P}_{\langle \Gamma \rangle^{\perp}}$.
  - `adjrec.israt` is `true` if the root r' is the class of a smooth rational curve on Y (that is, the chamber D' is outside of Nef_Y), whereas `adjrec.israt` is `false` if r' is not the class of a smooth rational curve on Y and hence the chamber D' is contained in Nef_Y.

When `adjrec.israt` is `true` and r' is the class of a smooth rational curve C' on Y, the record `adjrec` has the following item.

– `adjrec.split` is the pair of roots $[\widetilde{C}_1], [\widetilde{C}_2]$ of S_X that are the classes of the two irreducible components of $\pi^{-1}(C') = \widetilde{C}_1 + \widetilde{C}_2$.

When `adjrec.israt` is `false` and the adjacent chamber D' is contained in Nef_Y, the record `adjrec` has the following item.

– `adjrec.isnew` is `true` or `false`. If `adjrec.isnew` is true, then the chamber D' is G-equivalent to none of chambers that had been added to V_0 when we processed D', and hence D' was appended to V_0. If `adjrec.isnew` is false, the chamber D' is G-equivalent to a chamber that had been already added to V_0.

When `adjrec.israt` is `false` and `adjrec.isnew` is `true`, the record `adjrec` has the following item.

– `adjrec.positionInV0` indicates the position of V_0 at which D' is added, that is, the chamber D' is described by the record

$$\texttt{chamrec}' := \texttt{V0}[\texttt{adjrec.positionInV0}].$$

When `adjrec.israt` is `false` and `adjrec.isnew` is `false`, the record `adjrec` has the following item. Let D'' be the unique chamber in V_0 to which D' is G-equivalent.

– `adjrec.isomto` indicates the position of D'' in V_0.
– `adjrec.isomby` is the record `grec` of an element $g \in G$ such that $D'^g = D''$.

The subset $\mathcal{H}$ of the group G is expressed by a set `HHH` of records `grec`. **Caution.** The identity is not contained in `HHH`.

5. The file `Enrs.txt`

In the file `Enrs.txt`, we have a list `Enrs` of 182 reconds `Enr`. Each of these records corresponds to a $(\tau, \bar{\tau})$-generic Enriques surface Y listed in Table 1.1 of [BS], except for the cases Nos. 88 and 146 A record `Enr` in this list has the following contents.

- `Enr.no` is the number of the corresponding row in Table 1.1 of [BS].
- `Enr.typeR` is the ADE-type $\tau(R)$.
- `Enr.typeRbar` is the ADE-type $\tau(\overline{R})$.
- `Enr.typeRtilde` is the ADE-type $\tau(\widetilde{R})$.
- `Enr.exists` is `true` or `false`, and shows whether a $(\tau, \bar{\tau})$-generic Enriques surface exists or not.
- `Enr.cde` is the triple $[c_{(\tau,\bar{\tau})}, d_{(\tau,\bar{\tau})}, e_{(\tau,\bar{\tau})}]$.
- `Enr.ker` is the list of elements of the kernel of the natural homomorphism $\mathrm{aut}(X, \varepsilon) \to \mathrm{aut}(Y)$. Each element is a matrix in $\mathrm{O}^{\mathcal{P}}(S_X)$.
- `Enr.irecname` is the name if `irec` that is used in the computation.
- `Enr.SXrec` is a record describing the embeddings $S_Y(2) \hookrightarrow S_X \hookrightarrow L_{26}$ of lattices. See the subsection below for the details.
- `Enr.Autrec` is a record that describes the result of the computation in Section 6.1 of the paper [BS]. See the subsection below for the details.
- `Enr.Rats` is a record that describes the action of $\mathrm{aut}(Y)$ on the list $\mathcal{R}_{\mathrm{temp}}$ of smooth rational curves C on Y such that $[C]$ defines a wall of an $L_{26}/S_Y(2)$-chamber D belonging to `Enr.Autrec.V0`. See the subsection below for the details.

- `Enr.Ells` is a record that describes the action of $\mathrm{aut}(Y)$ on the list $\mathcal{E}_{\mathrm{temp}}$ of elliptic fibrations $\phi\colon Y \to \mathbb{P}^1$ such the ray $\mathbb{R}_{\ge 0}[F_\phi]$, where F_ϕ is a general fiber of ϕ, is contained in the closure $\overline{D}$ of an $L_{26}/S_Y(2)$-chamber D belonging to `Enr.Autrec.V0`. See the subsection below for the details. **Caution.** This item `Enr.Ells` is *not* provided if `Enr.exists` is `false`; that is, we calculate `Enr.Ells` only when the data `Enr.SXrec` is geometrically realizable.

5.1. **The record `Enr.SXrec`.** The record `Enr.SXrec` is the record describing the lattice S_X and the primitive embeddings

$$\iota\colon S_Y(2) \cong L_{10}(2) \hookrightarrow S_X \hookrightarrow L_{26}$$

of lattices. The record `Enr.SXrec` has the following items, many of which are just the copies of items of the record `irec` describing the primitive embedding $\iota\colon S_Y(2) \hookrightarrow L_{26}$. In the paper [BS], the orthogonal complement of $S_Y(2)$ in S_X is denoted by S_{X-}. In this note, we use Q to denote the lattice S_{X-}. The lattices L_{26}, S_X and Q are equipped with certain bases. The lattice $S_Y \cong L_{10}$ is equipped with the basis $\{e_1, \dots, e_{10}\}$.

- `SXrec.GramL26` is the Gram matrix of L_{26}.
- `SXrec.embSXL26` is the matrix M such that $v \mapsto vM$ is the embedding $S_X \hookrightarrow L_{26}$.
- `SXrec.embSYSX` is the matrix M' such that $v \mapsto vM'$ is the embedding $\pi^*\colon S_Y(2) \cong L_{10}(2) \hookrightarrow S_X$.
- `SXrec.embQSX` is the matrix M'' such that $v \mapsto vM''$ is the embedding $Q \hookrightarrow S_X$.
- `SXrec.GramSX` is the Gram matrix of S_X.
- `SXrec.configrats` is a list of roots $[\widetilde{C}_1], \dots, [\widetilde{C}_m]$ in S_X, where $\widetilde{C}_1, \dots, \widetilde{C}_m$ are distinct smooth rational curves on X such that the divisor $\sum \widetilde{C}_i$ is an ADE-configuration of type R and that $\sum \widetilde{C}_i$ is mapped isomorphically to a divisor on Y by $\pi\colon X \to Y$. The classes in this list together with the image S_{X+} of $\pi^*\colon S_Y(2) \hookrightarrow S_X$ generate S_X.
- `SXrec.walls` is the list of defining roots of the walls of the fixed $L_{26}/S_Y(2)$-chamber D_0.
- `SXrec.ampleY` is a primitive vector of $S_Y \cong L_{10}$ in the interior of the $L_{26}/S_Y(2)$-chamber D_0, which is an ample class of S_Y. This class is chosen in such a way that it is invariant under the action of the group $\mathrm{O}(L_{10}, D_0)$.
- `SXrec.m4vsInQ` is the list of (-4)-vectors in Q.
- `SXrec.codim2faces` is the list of non-ordered pairs $\{i, j\}$ such that the intersection of the ith wall and the jth wall in `SXrec.walls` is a face of codimension 2 of D_0.
- `SXrec.isotropicrays` is the list of primitive isotropic rays in the closure $\overline{D}_0$ of D_0.
- `SXrec.volumeindex` is $1_{\mathrm{BP}}/\mathrm{vol}(D_0)$.
- `SXrec.enrinvol` is the matrix presentation of the Enriques involution $\varepsilon \in \mathrm{O}^{\mathcal{P}}(S_X)$.

5.2. **The record `Enr.Autrec`.** The record `Enr.Autrec` is the record describing the results of the algorithm in Section 6.1 of the paper [BS] for the calculation of the action of $\mathrm{aut}(Y)$ on Nef_Y. This record `Enr.Autrec` is comprised of `Enr.Autrec.V0` and `Enr.Autrec.HHH` as are explained in Section 4.4.

5.3. **The record `Enr.Rats`.** The record `Rats` := `Enr.Rats` has the following items.

- `Rats.representatives` is the list of records `rrec` of smooth rational curves that are chosen as representatives of the orbit decomposition of $\mathcal{R}_{\mathrm{temp}}$ by the action of $\mathrm{aut}(Y)$.
- `Rats.Ratstemp` is the list of records describing elements of $\mathcal{R}_{\mathrm{temp}}$. For $C \in \mathcal{R}_{\mathrm{temp}}$, the corresponding record `temprrec` has the following items:
  - `temprrec.rat` is the record `rrec1` describing C.
  - `temprrec.rep` is the record `rrec2` describing the representative C' of the orbit containing C. Hence `rrec2` is a member of the list `Rats.representatives`.
  - `temprrec.by` is the record `grec` describing an automorphism $g \in \mathrm{aut}(Y)$ that maps C to C'.

5.4. **The record `Enr.Ells`.** This item `Enr.Ells` is provided only when `Enr.exists` is `true`. The record `Ells` := `Enr.Ells` has the following items.

- `Ells.representatives` is the list of records `ellfib` of elliptic fibrations $\rho\colon Y \to \mathbb{P}^1$ chosen as representatives of the orbit decomposition of $\mathcal{E}_{\mathrm{temp}}$ by the action of $\mathrm{aut}(Y)$. Each `ellfib` has the following item:
  - `ellfib.ell` is the primitive isotropic ray $f_\rho := [F_\rho]/2$, where F_ρ is a general fiber of $\rho\colon Y \to \mathbb{P}^1$.
  - `ellfib.reduciblefibers` describes the reducible fibers of the elliptic fibration $\rho\colon Y \to \mathbb{P}^1$. This record is a list of the records `fiberrec` corresponding to reducible fibers. Each `fiberrec` has the following items.
    - `fiberrec.adetype` is the ADE-type of the reducible fiber, which is one of the strings `"A1"`, `"A2"`, ..., `"E8"`. This string expresses the *affine* ADE-type of the reducible fiber.
    - `fiberrec.multiplicity` is the multiplicity m, which is 1 if the fiber has a reduced component, whereas $m = 2$ if the multiplicities of all components are even.
    - `fiberrec.irredcomps` is the list of records `rrec` that describe irreducible components of the fiber.
- `Ells.Ellstemp` is the list of records describing elements of $\mathcal{E}_{\mathrm{temp}}$. For $\phi \in \mathcal{E}_{\mathrm{temp}}$, the corresponding record `tempphirec` has the following items:
  - `tempphirec.ell` is the primitive isotropic ray $f_\phi := [F_\phi]/2$, where F_ϕ is a general fiber of ϕ.
  - `tempphirec.rep` is the primitive isotropic ray f_ρ of the representative $\rho\colon Y \to \mathbb{P}^1$ of the orbit pf ϕ under the action of $\mathrm{aut}(Y)$.
  - `tempphirec.by` is the record `grec` describing an automorphism $g \in \mathrm{aut}(Y)$ that maps f_ϕ to f_ρ.

6. The file `E6EnrRDPs.txt`

In the file `E6EnrRDPs.txt`, we have a list `E6EnrRDPs` of 750 records about the RDP-configurations on an (E_6, E_6)-generic Enriques surface Y. See Section 7.1 of the paper [BS]. Each record `RDPrec` in `E6EnrRDPs` describes an RDP-configuration

Γ obtained as $\Gamma(f)$ for some face f of the fundamental domain D_0. Each `RDPrec` has the following items.

- `RDPrec.position` is the position of `RDPrec` in the list `E6EnrRDPs`.
- `RDPrec.rats` is the list of smooth rational curves in the RDP-configuration Γ. Each member of `RDPrec.rats` is a record `rrec` explained in Section 4.3.
- `RDPrec.adetype` is the ADE-type of the RDP-configuration Γ.
- `RDPrec.isrepresentative` is `true` or `false`. This item being `true` means that Γ is chosen as a representative of the $\mathrm{aut}(Y)$-orbit of RDP-configurations containing Γ. In this case, the record `RDPrec` has the following two additional items.
  - `RDPrec.V0` and `RDPrec.HHH` are the result of Procedure 4.1 of the paper [BS] applied to the graph (V_Γ, E_Γ) and the group G_Γ.

  If `RDPrec.isrepresentative` is `false`, the record `RDPrec` has the following two additional items.
  - `RDPrec.isomto` is a pair $[i, j]$, which indicates the following. The ith RDP-configuration Γ' in `E6EnrRDPs` is the representative of the $\mathrm{aut}(Y)$-orbit containing Γ. Let `RDPrec`$'$ = `RDPrecs[i]` be the record describing the representative Γ'. The initial chamber D_0 of the graph (V_Γ, E_Γ) is mapped to the jth chamber D' in the list `RDPrec`$'$`.V0` by an element g of $\mathrm{aut}(Y)$ such that $\Gamma^g = \Gamma'$, which is described by `RDPrec.isomby` below.
  - `RDPrec.isomby` is a record `grec` explained in Section 4.2 that describes the automorphism $g \in \mathrm{aut}(Y)$ that maps D_0 to D' and Γ to Γ'.

References

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