AUTOMORPHISMS OF SUPERSINGULAR $K3$ SURFACES AND SALEM POLYNOMIALS: COMPUTATIONAL DATA

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We present the following data. These data are used for the proof of Theorems 1.2 and 1.3 in the paper

"Automorphisms of supersingular $K3$ surfaces and Salem polynomials".

We use the notation defined in this paper. Let X be a supersingular $K3$ surface in characteristic $p = \mathtt{p}$ with Artin invariant $\sigma = \mathtt{sigma}$.

- $\mathtt{GramSX}[\mathtt{p}, \mathtt{sigma}]$ is a Gram matrix of the lattice $\Lambda^-_{p,\sigma}$, which is isomorphic to S_X.
- $\mathtt{h0}[\mathtt{p}, \mathtt{sigma}]$ is a vector h_0 of $\Lambda^-_{p,\sigma}$ with $\langle h_0, h_0\rangle_\Lambda > 0$.
- $\mathtt{Rh0}[\mathtt{p}, \mathtt{sigma}]$ is the set $\mathcal{R}(h_0)$.
- $\mathtt{amplelist}[\mathtt{p}, \mathtt{sigma}]$ is an ample list of vectors $\boldsymbol{a} = [h_0, \rho_1, \dots, \rho_K]$. We identify $D(\boldsymbol{a})$ with $N(X)$ by a suitable isometry $\Lambda^-_{p,\sigma} \xrightarrow{\sim} S_X$.
- $\mathtt{sizeMs}[\mathtt{p}, \mathtt{sigma}]$ is the length l of the list $[M(h_1), \dots, M(h_l)]$ of matrix representations of double plane involutions $\tau(h_i) \in \mathrm{Aut}(X)$ whose product $\tau(h_1) \cdots \tau(h_l)$ is of irreducible Salem type.

For $\mathtt{i} = 1, \dots, l = \mathtt{sizeM}[\mathtt{p}, \mathtt{sigma}]$, we present the following data.

- $\mathtt{h}[\mathtt{p}, \mathtt{sigma}, \mathtt{i}]$ is the polarization h_i of degree 2 written as a row vector with respect to the basis of $\Lambda^-_{p,\sigma} \cong S_X$ that gives rise to the Gram matrix $\mathtt{GramSX}[\mathtt{p}, \mathtt{sigma}]$.
- $\mathtt{Exc}[\mathtt{p}, \mathtt{sigma}, \mathtt{i}]$ is the data of the smooth rational curves contracted by $\Phi_{h_i} : X \to \mathbb{P}^2$.
- $\mathtt{M}[\mathtt{p}, \mathtt{sigma}, \mathtt{i}]$ is the matrix representation $M(h_i)$ on S_X of the double plane involution $\tau(h_i) \in \mathrm{Aut}(X)$.

The data $\mathtt{Exc}[\mathtt{p}, \mathtt{sigma}, \mathtt{i}]$ is a list of lists of the form

$$[\mathtt{ADEtype}, \mathtt{Cs}],$$

each of which gives the information of a singular point P of B_{h_i}. $\mathtt{ADEtype}$ is the ADE-type of the singularity of P, and $\mathtt{Cs}$ is the list of the classes of smooth rational curves contracted to P by $\Phi_{h_i} : X \to \mathbb{P}^2$. These classes are sorted according to Figure 3.1 of the paper. If B_{h_i} is nonsingular, then $\mathtt{Exc}[\mathtt{p}, \mathtt{sigma}, \mathtt{i}]$ is the empty list [ ].

The following data gives the matrix representation M of the automorphism $\tau(h_1) \cdots \tau(h_l)$ of irreducible Salem type.

- $\mathtt{SalemM}[\mathtt{p}, \mathtt{sigma}]$ is the matrix

$$M := M(h_1) \cdots M(h_l),$$

where l is sizeMs[p, sigma].

- charpolSalemM[p, sigma] is the characteristic polynomial $\phi_M(t)$ of M, which is a Salem polynomial of degree 22.
- SalemNumb[p, sigma] is the real root of $\phi_M(t)$ larger than 1 in the floating-point number expression.

The files are divided as follows. (IS stands for IrreducibleSalem.)

- compdataIS0.txt: the data for p with $3 \leq p < 100$ and σ arbitrary.
- compdataIS1.txt: the data for p with $100 < p < 200$ and σ arbitrary.
- compdataIS2.txt: the data for p with $200 < p < 300$ and σ arbitrary.
- ...
- compdataIS79.txt: the data for p with $7900 < p \leq 7919$ and σ arbitrary.
- sigma10IS7.txt: the data for p with $7919 < p < 8000$ and $\sigma = 10$.
- sigma10IS8.txt: the data for p with $8000 < p < 9000$ and $\sigma = 10$.
- sigma10IS9.txt: the data for p with $9000 < p < 10000$ and $\sigma = 10$.
- ...
- sigma10IS17.txt: the data for p with $17000 << 17389$ and $\sigma = 10$.

These files are zipped in compdataIS.zip.