

BORCHERDS' METHOD FOR ENRIQUES SURFACES: COMPUTATIONAL DATA

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This note explains the contents of the computational data about the results of the paper [1] (joint work with Simon Brandhorst). The data is available at

<http://www.math.sci.hiroshima-u.ac.jp/~shimada/K3andEnriques.html>

in the text file `L10L26compdata.txt`. In this data, we use the Record-format of GAP [2].

We fix a basis e_1, \dots, e_{10} of L_{10} given in the paper [1].

- `stdNKconfigs` is the list of intersection matrices of the 7 Nikulin-Kondo configurations, sorted by the type I, II, \dots , VII.
- `GramL10` is the Gram matrix of L_{10} . (Hence the Gram matrix of $\mathbf{S} = L_{10}(2)$ is 2 times this matrix.)
- `a10` is the interior point $a_{10} = e_1^\vee + \dots + e_{10}^\vee$ of the Vinberg chamber V defined by $\langle x, e_i \rangle_{10} \geq 0$ for $i = 1, \dots, 10$.
- `Embs` is a list of 17 records, each of which contains the data for one of the 17 primitive embeddings of $\mathbf{S} = L_{10}(2)$ into L_{26} . The contents of each record in this list are explained below.

Let `irec` be a record corresponding to a primitive embedding $\iota: \mathbf{S} = L_{10}(2) \hookrightarrow L_{26}$. Recall that R_ι is the orthogonal complement of the image of ι in L_{26} . Then `irec` has the following contents.

- `irec.No` is the number of ι used in Tables of the paper [1].
- `irec.name` is the name of ι , which is one of the strings "12A", "12B", \dots , "96C", "infy".
- `irec.GramL26` is a Gram matrix of L_{26} with respect to a certain fixed basis. We use this basis for other data in this record `irec`.
- `irec.embS` is the 10×26 integer matrix M such that $v \mapsto vM$ gives the embedding $\iota: \mathbf{S} \hookrightarrow L_{26}$.
- `irec.embR` is the 16×26 integer matrix M' such that $v \mapsto vM'$ gives the embedding of R_ι into L_{26} with respect to a certain fixed basis of R_ι .
- `irec.GramR` is the Gram matrix of R_ι with respect to the fixed basis.
- `irec.rootsR` is the list of (-2) -vectors of R_ι .
- `irec.rootstypeR` is the ADE-type of (-2) -vectors of R_ι , which is a list of strings of "A1", "A2", \dots .

- `irec.m4R` is the number of (-4) -vectors of R_ι .
- `irec.ogR` is the order of $O(R_\iota)$.
- `irec.projS` is the 26×10 matrix P such that $v \mapsto vP$ gives the orthogonal projection $\text{pr}_S: L_{26} \rightarrow \mathbf{S}^\vee$.
- `irec.projR` is the 26×16 matrix P' such that $v \mapsto vP'$ gives the orthogonal projection $\text{pr}_R: L_{26} \rightarrow R_\iota^\vee$.
- `irec.weyl` is the Weyl vector $\mathbf{w} \in L_{26}$ such that $D := \iota_P^{-1}(C(\mathbf{w}))$ is an induced chamber in $\mathcal{P}(L_{10})$, where $C(\mathbf{w}) \subset \mathcal{P}(L_{26})$ is the Conway chamber corresponding to \mathbf{w} .
- `irec.weylprime` is another Weyl vector $\mathbf{w}' \in L_{26}$ such that $\langle \mathbf{w}, \mathbf{w}' \rangle_{26} = 1$, so that $a_{26} = 2\mathbf{w} + \mathbf{w}'$ is an interior point of $C(\mathbf{w})$.
- `irec.VR` is the list of vectors v of R_ι^\vee such that $\langle v, v \rangle_R = -1$, that is, the list of vectors in V_R in Table 2.1 of [1].
- `irec.walls` is the list of (-2) -vectors defining the walls of the induced chamber D . For the embedding of type `infty`, this list is a string `"infty"`.
- `irec.volindex` is $|\text{O}(L_{10}) \otimes \mathbb{F}_2| = 46998591897600$ divided by the number of Vinberg chambers contained in D . For the embedding of type `infty`, this list is a string `"infty"`.
- `irec.orderOL10D` is the order of the group of isometries of L_{10} that maps D to D , that is, the order of $\text{O}(L_{10}, D)$. For the embedding of type `infty`, this item is a string `"infty"`.
- `irec.generatorsOL10D` is a generating set of $\text{O}(L_{10}, D)$. Neither irredundancy nor minimality is proved for this generating set. For the embedding of type `infty`, this item is a string `"infty"`.
- `irec.interiorpt` is an interior point a_{10} of D that is fixed under the action of $\text{O}(L_{10}, D)$. For the embedding of type `infty`, this item is a string `"infty"`.
- `irec.NK` is the Nikulin-Kondo type of the induced chamber D , which is one of the strings `"I"`, `"II"`, `"...`, `"VII"`, or `"not corresponding to Nikulin-Kondo"`.
- `irec.NKisom` gives an isomorphism from the set of walls of D to the vertices of the standard Nikulin-Kondo configuration. Suppose that D is of Nikulin-Kondo type τ with n vertices, where $n = 12$ or 20 . Then `irec.NKisom` = $[\nu_1, \dots, \nu_n]$ means that the bijection that maps i th wall in `irec.walls` to the ν_i th vertex of the standard Nikulin-Kondo configuration of type τ in `stdNKconfigs` preserves the intersection matrix. If `irec.NK` is `"not corresponding to Nikulin-Kondo"`, then `irec.NKisom` is also `"not corresponding to Nikulin-Kondo"`.
- `irec.isomto`: If the induced chamber D is isomorphic to the induced chamber D' of a primitive embedding ι' of type different from `irec.name`, then

`irec.isomto` is the pair of the name of ι' and a bijection from the set of walls of D to the set of walls of D' that preserves the intersection matrix. The bijection is given in the same way as `irec.NKisom`. If there exists no such primitive embedding ι' , then `irec.isomto` is an empty list `[]`.

- `irec.wallrecs` is a list of records, each of which contains the data about a wall w of D . For the embedding of type `infty`, this list is a string `"infty"`. The contents of each record in this list are explained below.

Let `wrec` be a record in `irec.wallrecs` corresponding to a wall $w = D \cap (r)^\perp$ of the induced chamber $D := \iota_{\mathcal{P}}^{-1}(C(\mathbf{w}))$, where ι is not of type `infty`. Then `wrec` has the following contents. See the proof of Proposition 2.7 of [1] for notation.

- `wrec.r` is the (-2) -vector r of L_{10} that defines the wall $w = D \cap (r)^\perp$.
- `wrec.rlifts` is the list of (-2) -vectors \tilde{r} of L_{26} such that $\langle \mathbf{w}, \tilde{r} \rangle_{26} = 1$ and $\iota_{\mathcal{P}}^{-1}((\tilde{r})^\perp) = (r)^\perp$.
- `wrec.embQ` is the 17×26 integer matrix M such that $v \mapsto vM$ is the embedding $Q \hookrightarrow L_{26}$ under a certain fixed basis of Q .
- `wrec.GramQ` is the Gram matrix of Q with respect to the fixed basis of Q .
- `wrec.embQperp` is the 9×26 integer matrix M such that $v \mapsto vM$ is the embedding $Q^\perp \hookrightarrow L_{26}$ under a certain fixed basis of Q^\perp .
- `wrec.GramQperp` is the Gram matrix of Q^\perp with respect to the fixed basis of Q^\perp .
- `wrec.rootsQ` is the list of (-2) -vectors of Q .
- `wrec.rootstypeQ` is the ADE-type of the set of (-2) -vectors of Q .
- `wrec.Sigma` is the list of (-2) -vectors in Σ .
- `wrec.adjweyl` is the Weyl vector \mathbf{w}' corresponding to the Conway chamber $C(\mathbf{w}')$ such that $\iota_{\mathcal{P}}^{-1}(C(\mathbf{w}'))$ is the induced chamber adjacent to D across the wall $w = D \cap (r)^\perp$.
- `wrec.thegtilde` is the isometry $\tilde{g} \in O(L_{26}, \mathcal{P})$ such that \tilde{g} preserves the image of $\iota: \mathbf{S} \hookrightarrow L_{26}$ and that its restriction $\tilde{g}|_{\mathbf{S}}$ to \mathbf{S} maps D to the induced chamber adjacent to D across the wall $w = D \cap (r)^\perp$. The existence of this isometry proves Proposition 2.7 of [1].
- `wrec.theg` is the isometry $g = \tilde{g}|_{\mathbf{S}}$ of L_{10} . We can check that this isometry is equal to the reflection with respect to `wrec.r`.

REFERENCES

- [1] Simon Brandhorst and Ichiro Shimada. Borchers' method for Enriques surfaces, 2019. Preprint, <http://www.math.sci.hiroshima-u.ac.jp/~shimada/preprints.html>.
- [2] The GAP Group. *GAP - Groups, Algorithms, and Programming*. Version 4.8.6; 2016 (<http://www.gap-system.org>).

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