Destabilization/Stabilization of Diffusion Systems by Diffusion and Boundary Flux

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We study linear diagonal diffusion systems under linear non-diagonal Robin boundary conditions. “Non-diagonal” means that the flux of each component on the boundary depends linearly on the values of (possibly) all components consisting of the system. Our purpose is to show how the stability of the trivial solution depends on the diagonal diffusion matrix and the eigenvalues of the matrix (mass transfer matrix) which specifies the aforementioned linear dependence on the boundary. There are two simplifying situations where the stability and instability of the system are characterized in rather easy ways. One is the case in which the mass transfer matrix is symmetric, and the other is the case in which the diffusion rates are all equal. Investigations in these situations enable us to identify stable and unstable systems in terms of the stability/instability of the mass transfer matrix. Outside these two cases, to describe the critical eigenvalues by using the properties of the diffusion and mass transfer matrices is a subtle issue, and this leads to a Turing type instability. In contrast to closed reaction-diffusion systems, our system also exhibits a phenomenon called anti- Turing mechanism, in which originally unstable systems for equal diffusion rates are stabilized by biased (unequal) diffusion effects.