

A double bubble solution in a ternary system with inhibitory long range interaction

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We consider a ternary system of three constituents, a model motivated by the triblock copolymer theory. The free energy of the system consists of two parts: an interfacial energy coming from the boundaries separating the three constituents, and a longer range interaction energy that functions as an inhibitor to limit micro domain growth. We show that a perturbed double bubble exists as a stable solution of the system. Each bubble is occupied by one constituent. The third constituent fills the complement of the double bubble. Two techniques are developed. First one defines restricted classes of perturbed double bubbles. Each perturbed double bubble in a restricted class is obtained from a standard double bubble by a special perturbation. The second technique is the use of the so called internal variables. The advantage of the internal variables is that they are only subject to linear constraints, and perturbed double bubbles in each restricted class represented by internal variables are elements of a Hilbert space. A local minimizer of the free energy in each restricted class is found as a fixed point of a nonlinear equation. This perturbed double bubble satisfies three of the four equations for critical points of the free energy. The unsolved equation is the 120 degree angle condition at triple junction points. Perform another minimization among the local minimizers from all restricted classes. A minimum of minimizers emerges and solves all the equations for critical points.