Growth rate of a crystal surface with a co-rotating pair of spiral steps evolving by an eikonal-curvature flow

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In this talk we introduce a simple level set formulation for a co-rotating pair of spirals evolving by an eikonal-curvature flow, and a simple way to reconstruct a crystal surface with spiral steps proposed by screw dislocations. We apply this method to simulations of the evolution of a crystal surface by a co-rotating pair of spiral steps, and present some numerical results and observations on the growth rate and the critical distance of a co-rotating pair.

Burton, Cabrera and Frank proposed a theory of crystal growth with aid of screw dislocations in 1951. According to the theory a crystal surface with screw dislocations evolves in vertical direction of the surface with rotating spiral steps. The spiral steps evolve in the holizontal direction with an eikonal-curvature flow

$$V = v_\infty(1 - \rho_c \kappa), \tag{1}$$

where V is the velocity of the spiral steps, which are regarded as spiral curves on the plane, κ is the curvature of the spiral steps with the opposite direction of V, v_∞ and ρ_c are constants.

Burton et al also proposed some speculations on the growth rate of a crystal surface by a co-rotating pair of single spiral steps by approximating spirals with an Archimedean spiral $r = 2\rho_c\xi$, where (r, ξ) is the polar coordinate. They pointed out that, for a pair of co-rotating spiral centers a_1 and a_2, the growth rate (they call it "activity") of the pair is indistinguishable from that of a single spiral if $|a_1 - a_2| > 2\pi\rho_c$. If $|a_1 - a_2| \ll \rho_c$, then the activity of the pair should be twice of that of a single spiral. There is no estimate of the activity of the pair which lies between 1 and 2 times of the single one if $|a_1 - a_2| < 2\pi\rho_c$. The critical distance $\tilde{d}_c = 2\pi\rho_c$ is derived from the profile of the approximating Archimedean spiral.

In this talk we show more accurate critical distance of the co-rotating pair is farther than $2\pi\rho_c$ from numerical results. To obtain the critical distance we crudely estimate of the growth rate by a co-rotating pair, and numerically demonstrate that the growth rate R_p by the co-rotating pair is given as

$$R_p = \frac{2}{1 + |a_1 - a_2|\omega_1/(\pi\rho_c)} R_S \tag{2}$$

if $|a_1 - a_2| < \pi\rho_c/\omega_1$, where $\omega_1 = 0.330958061$ is the coefficient of the angle velocity $\omega = \omega_1 v_\infty/\rho_c$ for a single rotating spiral by (1) which is obtained by Ohara-Reid in 1973, and R_S is the growth rate by a single spiral step evolving with (1). From the view point of the growth rate we propose a new definition of the critical distance as the distance where $R_p = R_S$ in (2), and thus the more accurate critical distance is $d_c = \pi\rho_c/\omega_1$. Note that the approximation by the Archimedean spiral $r = 2\rho_c\xi$ yields that $\omega_1 = 1/2$. Thus our definition with the speculation by Burton et al yields the critical distance $\tilde{d}_c = 2\pi\rho_c$ by Burton et al.

This is the joint work with Y.-H. R. Tsai and Y. Giga.