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Dynamics of Two Equivalent Lanes Traffic Flow Model: Self-Organization of the Slow Lane and Fast Lane

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A simple two-lanes traffic flow model using cellular automaton is investigated. In this model, if the car density is set within a certain range, the following characteristic behavior of traffic flow is observed. i) The self-organization of the slow and fast lanes in spite of the symmetry between these two lanes. ii) The appearance of several branches and hysteresis in the relation between traffic flow and car density.

KEYWORDS: cluster, branch, symmetric state, asymmetric state

Recently, several traffic flow models have been proposed.[1-18] They succeeded in investigating the phenomena of one-lane traffic flow. However, the observed real systems contain more than two lanes. Hence, in order to investigate the multiple-lane effect, we simulate two-lanes traffic flow models which include the effects of lane changing.

We extend the one-lane model by Nagel and Schreckenberg[4] to the model with two lanes. The system consists of cars and lanes. The number of lanes is two and that of cars is varied as a control parameter. We number the cars in each lane in the order from the last to the first. Let x_n^i and v_n^i be the position of the front edge and the velocity of the ith car at time step n in one lane, respectively, and X_n^j and V_n^j be the position of the front edge and the velocity of the jth car in the other lane, respectively. The quantity d_n^i is the gap between the front edge of the ith car and the rear edge of the $i+1$th car i.e. $d_n^i = x_n^{i+1} - x_n^i - 1$ where 1 is the car length.

Velocity v_n^i takes one of the five discrete values from 1 to 5, and the dynamical rules for each lane are equivalent; additionally, the rules for changing lanes are set symmetrical between two lanes. The dynamics of the ith car from time step n to $n+1$ is determined by the set $\{x_n^i,\ v_n^i,\ v_n^{i+1},\ d_n^i,\ X_n^j,\ V_n^j\}$. Detailed rules are as follows where I), II) and III) show the dynamical rule in each lane and IV) shows the rule for lane changing;

I) When $v_n^i < d_n^i$; $x_{n+1}^i = x_n^i + v_n^i$ always holds, and $v_{n+1}^i = v_n^i + 1$ if $v_n^i < 5$, $v_{n+1}^i = v_n^i$ if $v_n^i = 5$.

II) When $v_n^i = d_n^i$; $x_{n+1}^i = x_n^i + v_n^i$ always holds. If $v_n^{i+1} > v_n^i$, the car accelerates as $v_{n+1}^i = v_n^i + 1$ with probability P (accelerating probability), or does not accelerate such that $v_{n+1}^i = v_n^i$ with probability $(1-P)$. For other cases $v_{n+1}^i = v_n^i$.

III) When $v_n^i > d_n^i$; $x_{n+1}^i = x_n^i + d_n^i$ always holds. If $v_n^{i+1} > v_n^i$ then $v_{n+1}^i = v_n^i$ and if $v_n^{i+1} = v_n^i$, $v_{n+1}^i = v_n^i - 1$. If $v_n^{i+1} < v_n^i$, $v_{n+1}^i = v_n^i - 1$ when $x_n^{i+1} = X_n^j$ or $X_n^j \le x_n^i < X_n^j + V_n^j$ holds for some jth car.

IV) When $v_n^i > d_n^i$ and $v_n^{i+1} < v_n^i$; similar to the case III), $x_{n+1}^i = x_n^i + d_n^i$ holds. If there is no jth car which satisfies the relation $x_n^{i+1} = X_n^j$ or $X_n^j \le x_n^i < X_n^j + V_n^j$, then the ith car moves into the next lane without changing velocity ($v_{n+1}^i = v_n^i$).

We simulate the two cases, (1) $P = 0$ (deterministic case) and (2) $P = 0.5$ (stochastic case). Here, we define the car density ρ as (number of occupied sites)/(system size) and flow f as $(\sum_i v^i)$/(system size) where system size means the total length of the road (=number of sites per lane)$\times$ 2 (=number of lanes).

First we discuss case (1). Due to the fact that the characteristic relation between the flow and the density and the non-trivial dynamics of the system appears for $0.1 \le \rho \le 0.2$, we focus on these densities. The boundary condition is set periodic and positions and velocities of cars are set random at the initial condition. Figure 1 is a typical spatio-temporal evolution for a stationary state where dots represent individual cars which trauel from the left to the right. Each stationary state is mainly composed of a free-flow-region and one type of clusters. Here a free-flow region is a region in which every car has the maximum velocity ($v_n^i = v_f = 5$). In contrast, a cluster is a region throughout which the (spatial) gap between two successive cars takes a constant value equivalent to their velocity v_c where $1 \le v_c \le 4$. Clusters are classified into 4 types according to v_c. In more detail 'accelerating region' ('decelerating region') exists between a free flow region and the front (rear) end of a cluster. In this region, cars accelerate from v_c to v_f (decelerate from v_f to v_c) and $v_n^{i+1} > v_n^i$ ($v_n^{i+1} < v_n^i$). Figure 2 shows the relation between density and flow for stationary states which are taken at the 50000th time-step after initial conditions with various car densities. In this diagram, some branches grow from the line $f = 5\rho$ for $0.1 < \rho < 0.2$. This means that flow is not determined as a unique function of car density. This result contradicts that of previous models,[1-13] but agrees with the observational reports of real systems.[20,21] The observational reports of real systems[20,21] show the time average of local flow and local density, contrary, we get the spatial average of them at a time. However, since the

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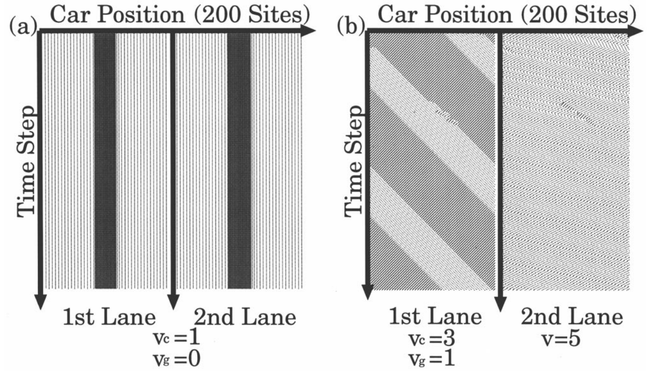

Fig. 1. Two typical spatio-temporal evolutions of the stationary state for P=0 (ρ = 0.17) where dots indicate individual cars which trauel from the left to the right. They are composed of a free flow region and one type of clusters in which each car trauels with v_c. (a) A symmetric state with v_c =-1. (b) An asymmetric state with v_c =-3.

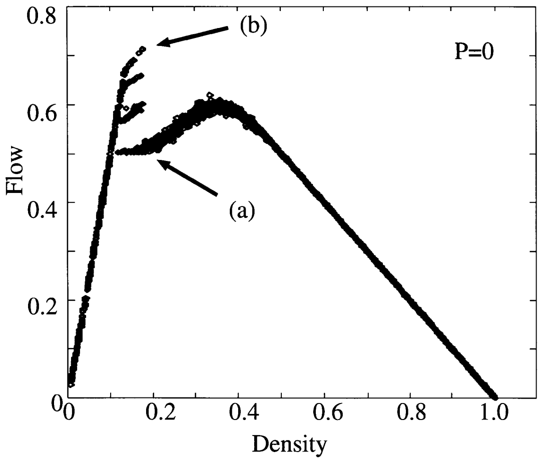

Fig. 2. The relationship between flow and density for P=0. Some branches exist for $0.1 \leq \rho \leq 0.2$.

system is in the stationary state, the time average of local flow and local density is supposed to be equivalent to their spatial average. Moreover each branch has its own spatio-temporal evolution of the corresponding system. For example, when the relationship between flow and density belongs to the branch (a) in Fig. 2, the system behaves like that in Fig. 1(a). This picture indicates that clusters composed of cars which run with $v_n^i = v_c = 1$ exist in both lanes in a symmetric manner. However, when the relation between flow and density belongs to the branch (b) in Fig. 2, the system behaves like that in Fig. 1(b). This picture indicates that one of these lanes include one type of cluster with $v_c = 3$, and the other lane is completely occupied by the free-flow region. The origin of the difference between the two systems, (a) and (b), lies in the initial distribution of cars. In this respect, even if the dynamical rules between two lanes are symmetrical, depending on the small difference in the initial conditions, two types of stationary states may appear. We call one of them 'symmetric state' (Fig. 1(a)) and the other 'asymmetric state' (Fig. 1(b)). Figure 3 shows the relationship between car densities and the average velocity difference between two lanes. It is natural to consider that the average velocity difference in the asymmetric state is larger than that in the symmetric state. Hence Fig. 3 indicates that symmetric and asymmetric

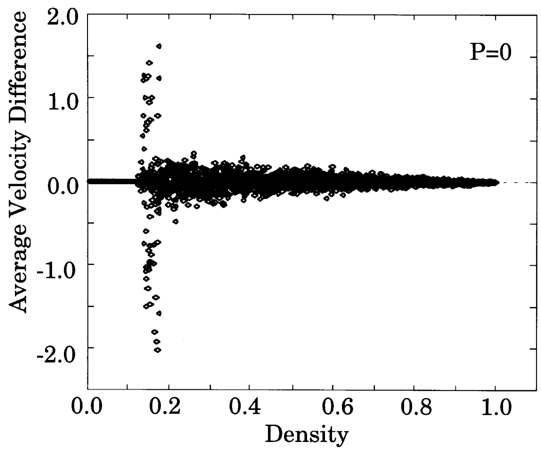

Fig. 3. The relationship between average velocity difference between two lanes and car density for P=0.

states coexist for densities $0.1 \leq \rho \leq 0.2$.

On investigating our simulation results in further detail, we obtain the following facts. In Fig. 1, each cluster has a unique group velocity v_g which is given as $v_g = \frac{v_c - 1}{2}$, where the group velocity is defined as the average velocity of the front edge of a cluster. Similarly, if a free-flow region steadily exists in front of a cluster, the car gap d in the free-flow region is given by $d = 2v_f - v_c$. For $0.1 \leq \rho \leq 0.2$, the relationship between the density ρ and the flow f for each state is nicely fitted by the following experimental equations, (Fig. 4).

i) When the symmetric state is realized,

$$f = \frac{1}{2} + v_g \rho. \tag{1}$$

ii) When the asymmetric state is realized,

$$f = \frac{3}{4} - \frac{1}{2(v_f + 1)} + v_g(\rho - \frac{1}{2(v_f + 1)}). \tag{2}$$

In both cases i) and ii), considering the fact that v_g sensitively depends on the initial configuration and is not uniquely determined by ρ, f is a non-unique function of ρ. Moreover, when we add cars one by one at random positions such that the car density increases from 0.1 to 0.2, the flow changes along several branches except in the lowest branch in Fig. 2. On the other hand, when we remove cars such that the car density decreases from 0.25 to 0.1, the $\rho - f$ relation varies along the lowest branch of Fig. 2. This means that hysteresis exists in the relationship between the flow and the density. (Fig. 5)

Next we discuss the manner in which the symmetric state and the asymmetric state appear and which type of clusters finally survives in the system. In order to discuss these, we consider the dynamics of cluster in the relaxation process of the system. Consider the situation in which two isolated clusters named 'the front cluster' and 'the rear cluster' are in the same lane and a free-flow region exist in-between, where v_c of these clusters are named $v_{c(fr)}$ and $v_{c(re)}$ respectively. In the free-flow region in front of the front cluster the car gap is given by $d_f = 2v_f - v_{c(fr)}$. This means the number of cars which pass a point in this free-flow region per unit time is $\frac{v_f}{d_f + 1} = \frac{v_f}{2v_f - v_{c(fr)} + 1}$. This is equivalent to the escape rate of cars from the front cluster. Similarly the escape

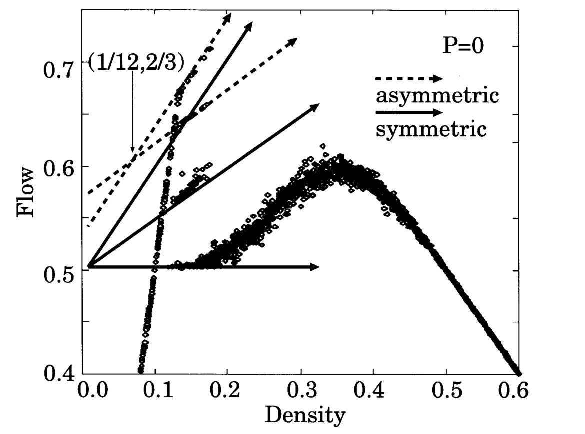

Fig. 4. The relationship between flow and density for P=0. Each branch is fitted by either eq. (1) or (2) depending on its state symmetry.

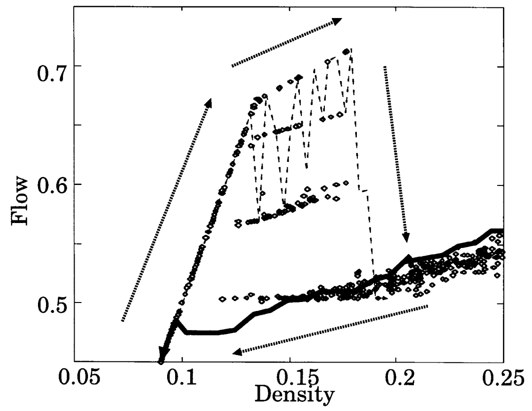

Fig. 5. Hysteresis exists in the relation between flow and car density.

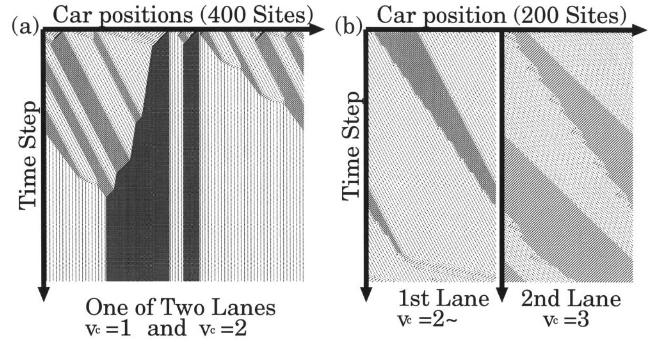

Fig. 6. Spatio-temporal evolutions of relaxation processes. (a) The interactions of some clusters which appear in a lane ($\rho = 0.22$). (b) The interactions of two clusters which appear in different lanes occupy regions neighboring each other ($\rho = 0.17$).

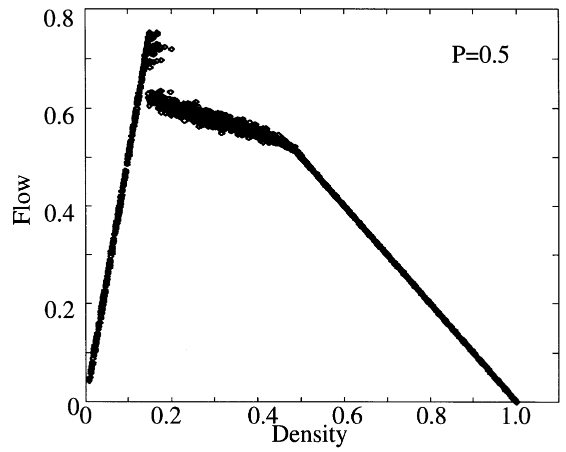

Fig. 7. The relationship between flow and car density for P=0.5. Similar to the case of P=0, the flow is not a unique function of the density.

rate of cars from the rear cluster is $\frac{v_f}{2v_f - v_{c(re)}+1}$ which is equivalent to the joining rate of cars into the front cluster. From these facts, if $v_{c(fr)} > v_{c(re)}$, the number of outgoing cars from the front cluster is larger than that of cars joining the cluster per unit time. Consequently, the length of the front cluster continues to decrease and finally disappears. Similarly if $v_{c(fr)} < v_{c(re)}$ the front cluster increases, and if $v_{c(fr)} = v_{c(re)}$ the length of the front cluster remains unchanged. When $v_{c(fr)} < v_{c(re)}$, the rear cluster occasionally catches up with the front cluster before it disappears. Then, from the dynamical rule, the front-most car of the rear cluster must decelerate from $v_{c(fr)}$ to $v_{c(re)}$ when the car cannot change lanes. Hence, the cars in the rear cluster will be incorporated into the front cluster and finally the rear cluster disappears. Figure 6(a) indicates these processes. Second, consider the situation where two clusters in different lanes occupy regions neighboring each other. When v_c of these clusters are different, the front edge of the faster cluster moves faster than that of the slower cluster. However, their rear edges tend to correspond with each other as a results of lane changing of individual cars near the rear-most of these clusters. This means that, due to the contact of two clusters in different lanes, the car number of the slower cluster decreases and that of the faster cluster increases. Hence the slower cluster becomes shorter and shorter and finally disappears (Fig. 6(b)). In this way asymmetric states are spontaneously formed. On the other hand if v_c of the final clusters in each lane are identical, this state is the symmetric state and remains stable.

Finally we consider the situation where a new cluster spontaneously arises by one of the following mechanisms. One is, when a cluster disappears, the accelerating region and the decelerating region come into contact with one another. Then, a seed of a new cluster is formed by the cars in these regions which have velocities larger than the car velocity of the cluster that just disappeared. From this seed, a new cluster with a group velocity faster than before is occasionally produced when the incoming flux of cars is sufficient. The other mechanism is, when cars enter a lane from the other lane which includes clusters, the local car density of the former lane increases. Then a seed of a new cluster is occasionally created in this position. This seed may not survive to grow up to a cluster if the incoming flux of cars is not sufficient. Through the above dynamics of clusters, the system is relaxed to either the symmetric state in which both lanes include the same type of cluster, or, to the asymmetric state in which only one lane contains a type of clusters and the other lane is filled with the free flow region.

Finally, we study the cases where stochastic effects are introduced into the system. For $P = 0.5$, the behavior of the system is qualitatively the same as that in the deterministic case ($P = 0$) (Figs. 7 and 8). This means that

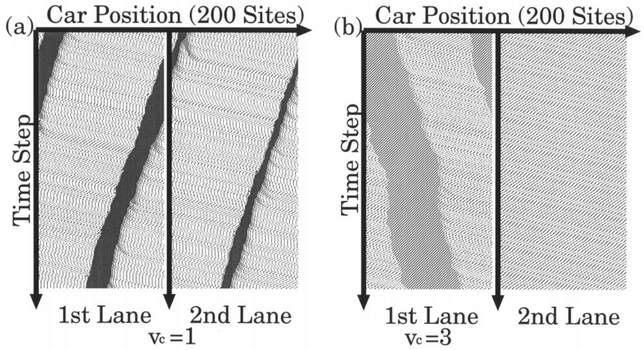

Fig. 8. Two typical spatio-temporal evolutions of the stationary states for P=0.5 (ρ =-0.16). (a) A symmetric state with v_c =-1. (b) An asymmetric state with v_c =-3.

the deterministic model catches the intrinsic dynamics of two-lane traffic flow. However several aspects need further discussion with regard to the stochastic effects. For example, if $P = 0$, v_g has positive values (Fig. 1) although the observation of real systems simulation of $P = 0.5$ show v_g has negative value (Fig. 8).

To summarize, the following novel properties for traffic flow are realized using a two-lanes traffic flow model. There are two types of stationary states, (a) symmetric state composed of two lanes including same-type clusters in a symmetric manner and (b) asymmetric state composed of one lane containing clusters and the other lane filled with the free flow region. Depending on the initial condition, one of the above two states ((a) and (b)) appears for the density $0.1 \leq \rho \leq 0.2$, which means the flow of the stationary state is not a unique function of density. Moreover, hysteresis exists in the relation between flow and density.

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