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Cellular automaton rule 184 + + C. A simple model for the complex dynamics of various particles flow

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Abstract

A cellular automaton named Rule 184 + + C is proposed as a meta-model to investigate the flow of various complex particles. In this model, unlike the granular pipe flow and the traffic flow, not only the free-jam phase transition but also the free-intermediate, the intermediate-jam, and the dilute-dense phase transitions appear. Moreover, the freezing phenomena appear if the system contains two types of different particles. © 1999 Published by Elsevier Science B.V. All rights reserved.

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Recently, the flows of materials which consist of numerous discrete elements, for example the granular pipe flow, the traffic flow, and so on are investigated analytically, experimentally, and numerically [2–16]. They succeeded to explain phenomena in these systems, e.g. the free-jam phase transition and the $1/f^\alpha$ fluctuation of the local density [4,9,13,17]. However, it is a fact that the behavior of the granular pipe flow depends on materials filled in the pipe [13,17]. For example, when the long range interactions like the Coulomb force, the inhomogeneity of softness, or the form of materials is taken into account, the system is expected to behave in a more complex manner. Now, to discuss such complex flows of various particles, we propose a simple

meta-model which we name cellular automaton (CA) Rule 184 + + C.

The dynamics of Rule 184 + + C is based on that of Rule 184. The Rule 184 is taken as one of the simplest models of the traffic and the granular flow. Here, in addition to the Rule 184 dynamics, as the + C rules, we employ a set of simple rules for the velocity change of individual particles. The dynamics of each particle is described by the equations

$$v_{n+1}^i = F(v_n^i, v_n^{i+1}, d_n^i), \quad (1)$$

$$x_{n+1}^i = x_n^i + v_{n+1}^i, \quad (2)$$

where we number the particles i from the upper part of the traffic stream to the downward. The quantities x_n^i and v_n^i are the position and the velocity of the i th particle at time step n , and d_n^i is the number of

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empty sites between i th and $(i+1)$ th particle. The function $F(v_n^i, v_n^{i+1}, d_n^i)$ and the velocity v_n^i take 0 or 1, whereas $F(*)$ obeys the following rules;

I) When $d_n^i > 1$, $F(v_n^i, v_n^{i+1}, d_n^i) = 1$ always holds, and when $d_n^i = 0$, $F(v_n^i, v_n^{i+1}, 0) = 0$ always holds.

II) When $d_n^i = 1$, $F(v_n^i, v_n^{i+1}, d_n^i = 1)$ takes 0 or 1 which depends on the type of the particle at i . Since v_n^i takes 0 or 1, the combination of (v_n^i, v_n^{i+1}) takes one of the following four patterns, (0,0), (0,1), (1,0), (1,1). For each combination, F takes the value 0 or 1. Hence, 16 types of rules for $F(v_n^i, v_n^{i+1}, 1)$ (from $\{F(0,0,1) = 0, F(0,1,1) = 0, F(1,0,1) = 0, F(1,1,1) = 0\}$ to $\{F(0,0,1) = 1, F(0,1,1) = 1, F(1,0,1) = 1, F(1,1,1) = 1\}$) are considered. In other words, we define 16 types of particles according to the function $F(v_n^i, v_n^{i+1}, 1)$. Now, we name the types of particles using the rule number C which is defined like Wolfram's method [1],

$$C = 2^0 F_C(0,0,1) + 2^1 F_C(0,1,1) + 2^2 F_C(1,0,1) + 2^3 F_C(1,1,1). \quad (3)$$

If the type of all particles is $C = 15$, the dynamics is same as CA Rule 184. The boundary condition is set periodic and the positions and the velocities of particles are set random at initial conditions. Hereafter, for the first, simulation results for pure systems in which all particles have same C are introduced. Also the statistical and the dynamical properties for each stationary state are discussed. Second, the simulation of mixed systems in which various C particles coexist are discussed.

In Fig. 1 are the typical fundamental diagrams which are the relations between the particle density of the system ρ and the flow f for each C . Here, ρ is defined as (the number of the occupied sites by particles)/(the total number of sites), and f is defined as $\langle \sum_i v_n^i \rangle / (\text{the total number of sites})$ where $\langle \dots \rangle$ means the time average. These fundamental diagrams are classified into following three types: (i) the two phases type (2P-type) where $C = 0, 4, 5, 6, 8, 10, 12, 13, 14$, or 15, (ii) the three phases type (3P-type) where $C = 2, 9$, or 11, and (iii) the four phases type (4P-type) where $C = 1, 3$, or 7. The property of the steady state for the 2P-type is following; When ρ is low, all particles move at each time step, while slugs appear in the system if ρ is higher than a critical value. Here, the term 'slug' means an

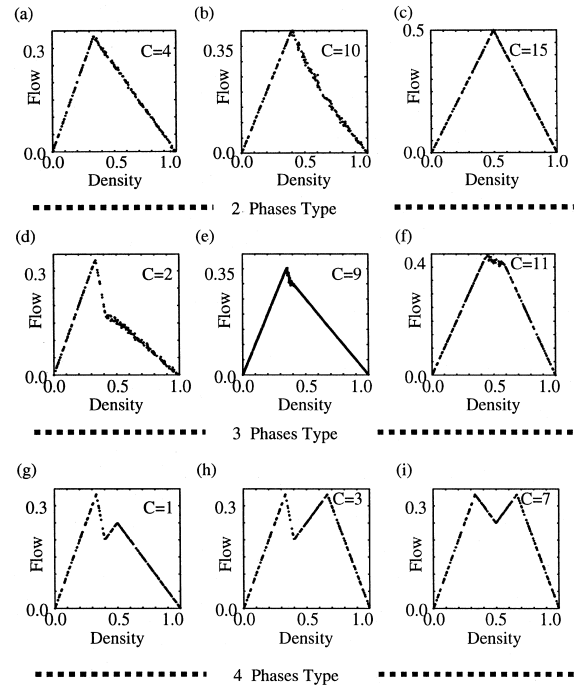


Fig. 1. Typical fundamental diagrams for each C . There are three types of fundamental diagrams, (a),(b),(c) 2P-type, (d),(e),(f) 3P-type, and (g),(h),(i) 4P-type.

array of particles which do not move i.e., $v_n^j = 0$ $v_n^{j+1} = 0 \dots$. Generally, slugs move backward in the traffic stream keeping the internal spatial pattern of them invariant. Each of them is kept by the balance between the incoming free-flow particles from the upward and the outgoing free-flow particles to the downward. As such, with ρ increases, the phases transition from the 'free-flow state' (without slug) to the 'jam-flow state' (with slugs) occurs. This property is qualitatively same as known results of recent traffic and granular flow models [2–13]. In particular, the dynamics of the pure particle systems with $C = 12$ ($F_{12}(0,0,1) = 0, F_{12}(0,1,1) = 0, F_{12}(1,0,1) = 1, F_{12}(1,1,1) = 1$) is equivalent to the dynamics of the deterministic traffic flow model proposed by Takayasu and Takayasu [9]. Different from such systems, the 3P-type systems include the parameter region of the 'intermediate-flow state'. At the intermediate ρ values between those of the free-flow state and of the jam-flow state, the third state which is different from the former two states, takes place as shown in Fig. 2 (b) and (e). Among the 3P-type

systems, we discuss the pure systems with $C = 11$ and $C = 9$. Fig. 2 (a) (b) and (c) show the space-time evolutions of the stationary states of the pure systems with $C = 11$ ($F_{11}(0,0,1) = 1$, $F_{11}(0,1,1) = 1$, $F_{11}(1,0,1) = 0$, $F_{11}(1,1,1) = 1$). They respectively correspond to (a) the free-flow state, (b) the intermediate-flow state and (c) the jam-flow state. Here, dots represent individual particles where black dots indicate $v = 0$ particles and gray dots are $v = 1$ particles. The behaviors of particles in (a) and (c) of Fig. 2 are qualitatively same as the space-time evolutions of the 2P-type systems. In the free-flow state, more than two successive empty sites appear in front of all particles because the particle density is low. In the jam-flow state, slugs appear and survive stably because the particle density is high. Different from these two kinds of states, in the intermediated-flow state, stable slugs and more than two successive empty sites coexist (Fig. 2 (b)). Fig. 2 (d) (e) and (f) show the space-time evolutions of the stationary states of the system with the $C = 9$ particles ($F_9(0,0,1) = 1$, $F_9(0,1,1) = 0$, $F_9(1,0,1) = 0$, $F_9(1,1,1) = 1$). There are two types of slugs. Isolated slugs consist of only one particle j with $v_n^j = 0$, whereas large slugs consist of more than two particles with $v_n^j = 0$, $v_n^{j+1} = 0 \dots$. Moreover the backward propagation velocity of large slugs is slower than that of isolated ones. In the jam-flow state (Fig. 2(f)), some large slugs remain stable in the system. In the intermediate-flow state, however, large slugs are unstable and repeat creation and annihilation irregularly in space and time (Fig. 2(e)). The intermediate-flows are new flow phases which have not been reported in the observation of real traffic flow or the granular flow. However, the spatio-temporal patterns like Fig. 2(e) and (f) are similar to these of stochastic model [7] which describes the realistic traffic flow.

Next, we discuss the properties of the 4P-type systems. As an example, the system with $C = 3$ particles ($F_3(0,0,1) = 1$, $F_3(0,1,1) = 1$, $F_3(1,0,1) = 0$, $F_3(1,1,1) = 0$) is considered. In Fig. 3 are typical space-time evolutions for several typical densities. When density ρ is low ($0 < \rho < \frac{1}{3}$), the free-flow state is realized, and when ρ increases ($\frac{1}{3} < \rho < \frac{2}{5}$) some slugs emerge, which means the free-jam transition takes place like in the 2P-type systems (Fig. 3 (a),(b)). Within slugs of the latter ρ region the gap

d_n^j between particle j and $j + 1$ is not zero, but repeats $d_n^j = 1$ and $d_{n+1}^j = 2$ by turn. Moreover, the spatial patterns in these slugs are periodic with the unit $v_n^j = 1$, $v_n^{j+1} = 0$, $d_n^j = 1$ and $d_n^{j+1} = 2$. In other words, these slugs are more dilute than what we see in the 2P-type and the 3P-type systems wherein gaps d are all 0. Thus, we name these slugs ‘dilute slugs’ and this state the ‘dilute jam-flow state’. When $\rho = \frac{2}{5}$, the system is completely filled with dilute slugs. If ρ increases more ($\frac{2}{5} < \rho < \frac{2}{3}$), unlike the 2P-type systems, different type of slugs from the dilute slugs appear (Fig. 3 (c)). Spatial pattern of these slugs is periodic with the unit $v_n^j = 1$ and $v_n^{j+1} = 0$ similarly with the dilute slug. However, in this case, the gaps in front of particles j and $j + 1$ repeat the combinations ($d_n^j = 0$, $d_n^{j+1} = 1$) and ($d_{n+1}^j = 1$, $d_{n+1}^{j+1} = 0$) by turn. Moreover, the direction of movement of these slugs is downward, and these slugs are surrounded by dilute slugs. Now, we name these unusual slugs ‘advancing slugs’ and this state the ‘advancing jam-flow state’. Moreover, in the advancing jam-flow state, the flow increases in proportional to ρ , in which sense the advancing jam-flow state is similar to the free-flow state. When the density increases more ($\frac{2}{3} < \rho$), slugs in which the gaps d_n^i are zero appear (Fig. 3(d)), and the flow turns again to a decrease function of ρ . We call this state the ‘hard jam-flow state’. As such, with the increase of ρ , three phase transitions, the free–dilute–jam, the dilute–jam–advancing–jam, and the advancing–jam–hard–jam transitions appear. The characters of the first and the third are like what we see in the 2P-type system because the flow sharply turns from the increasing function of ρ to a decreasing function at the transition point. So both of them are, in a wide sense, the free–jam transitions. On the other hand, the second is the transition between the low density (dilute) regime and the high density (dense) regime. The similar transitions also appears for $C = 1$ and $C = 7$ systems. However, these transitions have not been observed in the real traffic flow nor in the granular flow. They are expected to appear if the interaction between particles are more complex than that between drivers or between granular particles.

It is noted the 3P-type systems and the 4P-type systems, respectively, have common rules which hold among each of them. The 3P-type systems share the rules $F_C(0,0,1) = F_C(1,1,1)$ and $F_C(1,0,1) = 0$. These

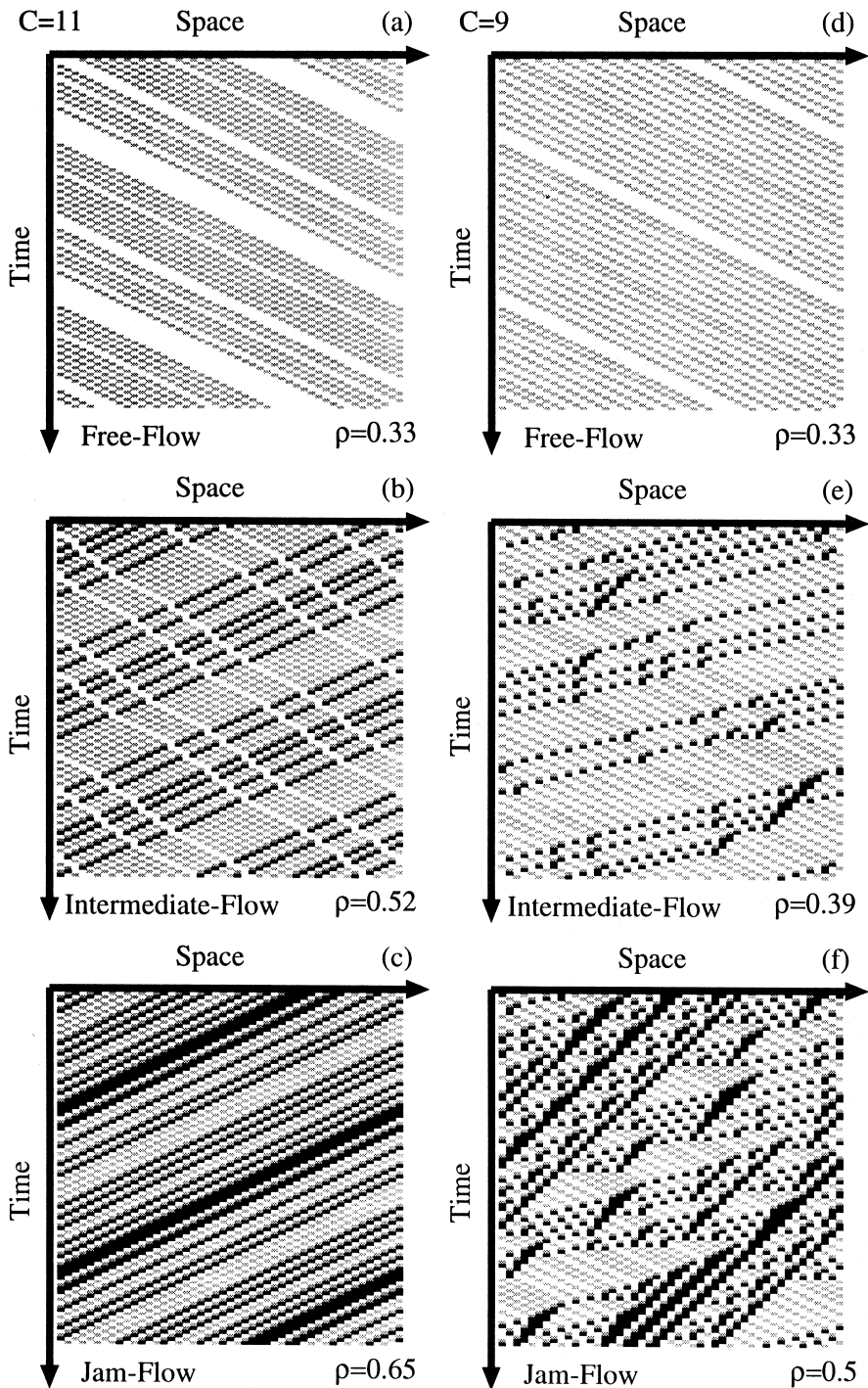


Fig. 2. Space-time evolutions of the stationary states of $C = 11$ particles systems ((a),(b),(c)), and $C = 9$ ((d),(e),(f)), where black dots means $v = 0$ particles and gray dots means $v = 1$ particles. They indicates, respectively, the free-flow ((a) and (d)), the intermediate-flow ((b) and (e)), and the jam-flow ((c) and (f)).

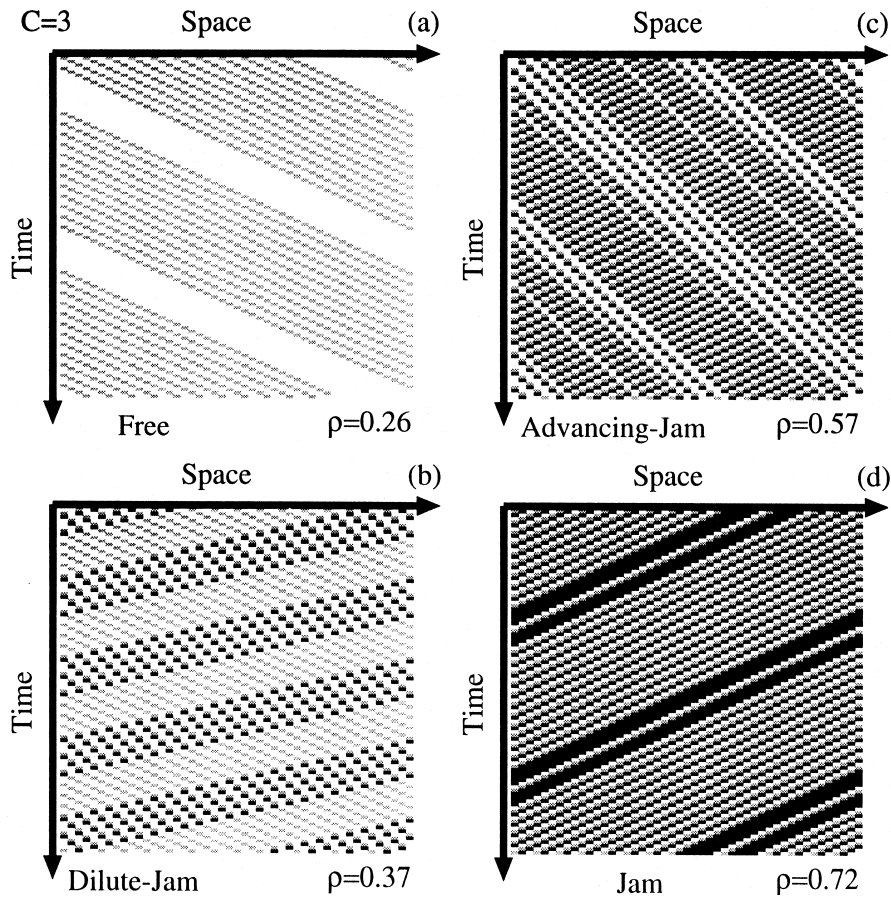


Fig. 3. The space-time evolutions of the stationary states of $C = 3$ particles systems, respectively, (a) the free-flow, (b) the dilute jam-flow, (c) the advancing jam-flow, and (d) the hard jam-flow.

rules mean that when a particle comes close to the preceding particle, repulsive force works between the two. On the other hand, the 4P-type systems share the rules, $F_C(0,0,1) = 1$ and $F_C(1,1,1) = 0$. Here, the rule $F_C(0,0,1) = 1$ indicates that effective attractive force works on a particle when this particle and the preceding one are close each other and they are at a standstill (i.e. $v_n^i = v_n^{i+1} = 0$). Similarly, $F_C(1,1,1) = 0$ indicates that effective resistance acts on the rear particle when this particle and the preceding one are moving (i.e. $v_n^i = v_n^{i+1} = 1$). According to such effective forces, above mentioned several flow phases are realized.

So far we have discussed the statistical aspects of our system. In the next, we consider the dynamical properties of the above systems. For this purpose, apart from the classification of the system according

to the fundamental diagrams, the space-time evolutions are divided into three types. i) The regular flow regime ($C = 0, 2, 3, 5, 8, 10, 11, 12, 14$, or 15); where space-time evolution is regular and the flow f is constant with time (As shown in Fig. 2 (a) (b) (c), and Fig. 3). ii) The oscillatory flow regime ($C = 1, 4, 7$, or 13); where f oscillates with time. For example in the $C = 7$ particle system, the velocities of individual particles in dilute slugs synchronize each other to oscillate between 0 and 1 (Fig. 4 (a)). Therefore, f oscillates with large amplitude (Fig. 4 (b)). iii) The chaotic flow regime ($C = 6$, or 9); where the space time evolution of particles and the time evolution of f are chaotic. (Examples are shown in Fig. 2 (e), (f), and Fig. 5(a).) In particular, in the $C = 9$ particles system, the flow has $1/f$ fluctuation near the critical density ($\rho \sim 0.38$) of the phase

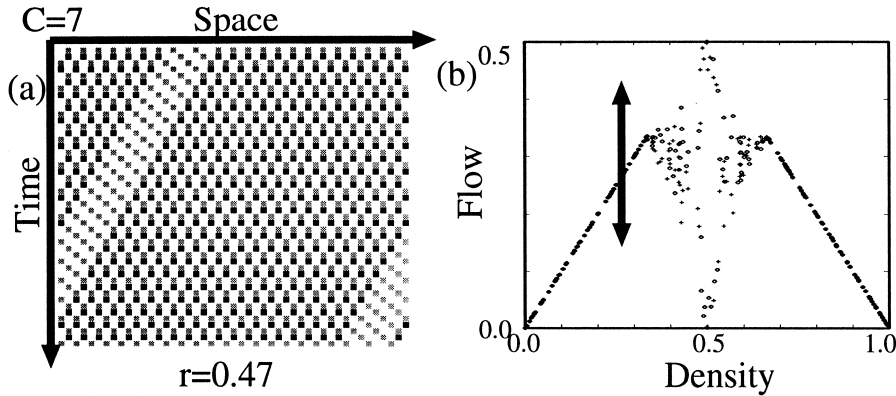


Fig. 4. (a) Space-time evolutions of the stationary states of $C = 7$ particles systems. (b) Fundamental diagram of $C = 7$ particles system at even time and odd time.

transition between the intermediate-flow state and the jam-flow state (Fig. 5(b)). Both type of particles (i.e. $C = 6$ and $C = 9$) share the symmetric dynamics $F_C(0,0,1) = F_C(1,1,1) \neq F_C(0,1,1) = F_C(1,0,1)$. In other words, only such systems that contain particles with symmetric rules behave chaotic if the system is pure.

Finally, we focus the mixed systems in which two values of C particles are mixed and compare the behavior of them to those of the pure systems. Fig. 6 are the typical fundamental diagrams of two types of pure systems and that of mixed systems of these two types of particles. Here, the ratio of two types of particles is 1:1. Almost all the cases, the relations like Fig. 6 (a) are realized. However, the relations like Fig. 6 (b) also are realized for some cases. Here, the flow of the mixed systems is smaller than that of respective pure particles systems. To discuss them, as an example, we consider mixed systems with

$C = 2$ and $C = 4$ particles (For $C = 2$: $F_2(0,0,1) = 0$, $F_2(0,1,1) = 1$, $F_2(1,0,1) = 0$, $F_2(1,1,1) = 0$. For $C = 4$: $F_4(0,0,1) = 0$, $F_4(0,1,1) = 0$, $F_4(1,0,1) = 1$, $F_4(1,1,1) = 0$). Fig. 7 (a) is a fundamental diagram of the almost pure $C = 2$ particles system except one $C = 4$ particle inside it. Compared to the pure $C = 2$ particles system, the flow is drastically little, and in particular, no flow for $\rho \geq 0.5$. The origin of such behavior is following. When ρ is sufficiently large, that is, most of the gaps d_n^i between successive particles are not larger than 1, the dynamics of each particle mainly obeys the function $F_C(v_n^i, v_n^{i+1}, 1)$. Once the $C = 4$ particle stops, this particle remains stationary if d_n^i remains not larger than 1. Moreover, in such d_n^i , $C = 2$ particle remains stationary when the preceding particle does not move. In other words, a $C = 4$ particle works as the coagulant of $C = 2$ particles, and change the characters of the whole system. Such relation appears also in different pairs

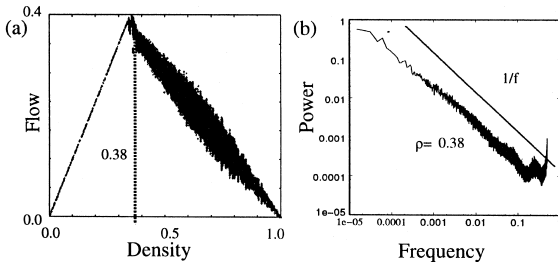


Fig. 5. (a) Fundamental diagram of $C = 9$ particles system. (b) Power spectrum of the flow fluctuation of $C = 9$ particles system near the critical density ($\rho \sim 0.38$) of intermediate-jam transition.

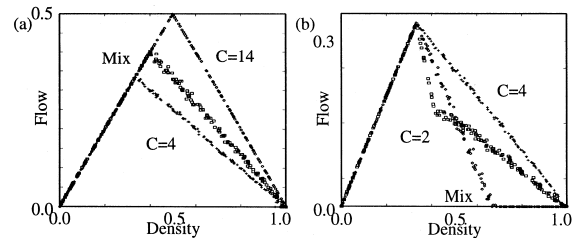


Fig. 6. Typical fundamental diagrams of respective two pure particles systems and the mixed particles systems. The ratio of particles is 1:1. (a) Normal type. (b) Decreasing (by the mixing) type.

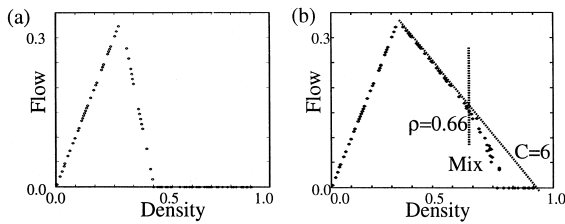


Fig. 7. Fundamental diagrams of the 'almost pure' systems; (a) $C = 2$ particles system and the $C = 6$ particles system with one $C = 4$ particle inside of each. The dotted line across the ρ axis with $\rho = 0.66$.

of particles e.g., $C = 6$ and $C = 4$ for $\rho > 0.66$ (Fig. 7 (b)), and $C = 2$ and $C = 12$. The mechanism of the freezing phenomena in these systems is qualitatively same as the above. Moreover, $C = 4$ and $C = 12$ particles, both of which work as the coagulant in the above mixed systems, share the common rules $F_C(0,0,1) = 0$ and $F_C(0,1,1) = 0$.

To summarize, various properties of complex materials flows are realized using a meta-model named CA Rule $184 + + C$. Unlike the previous CA models of the granular particles flow or the traffic flow, two types of novel relations between the density and the flow are realized in some types of particles systems. One is realized in the three phases type systems in which the free-intermediate and the intermediate-jam phase transitions occur with the increase of particle density. The other is observed in the four phases type systems in which not only the free-jam but also the dilute-dense transitions are realized. When these systems are classified from another point of view, there are three types of flow regimes: the regular flow regime, the oscillatory flow regime, and the chaotic flow regime. The particles in the chaotic flow systems share the symmetric rules. The relationship between such symmetric rule and the chaotic behavior remains to be studied in the future. Moreover, when two types of particles are mixed in a system, one may work as a coagulant of the other. In other words, the characters of pure particles systems are drastically changed by the addition of only a few number of different type of particles. The present CA rules are straight extension

of the rule 184. Hence, it is expected that some of $F_C(*)$ particle systems describe the behaviors of traffic flow or granular flow more faithfully than the rule 184 whereas some of $F_C(*)$ will not. In practice, behaviors of a chaotic flow system ($C = 9$) are similar to those of stochastic model for the real traffic flow [7], and the dynamics of the systems with $C = 12$ is equivalent to that of the deterministic traffic flow model [9]. The more systematic discussion on which of above $F_C(*)$ s are suitable for the model of real traffic flow is one of the future issues.

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