

第 8 回広島整数論集会



下記の日程で研究集会を開催します。

2009年7月21日(火) – 7月24日(金)
広島大学理学部 B 棟 B 707

本研究集会は、日本学術振興会・先端研究拠点事業「数論幾何・モチーフ理論・ガロア理論の新展開と、その社会的実用」(コーディネーター 松本眞)、科研費基盤研究(A)「数論・幾何の新展開：数論的トポロジー、圏論的数論幾何、アルゴリズム」(代表者 松本眞)からの補助を受けております。

プログラムの変更・講演のアブストラクト・会場までの交通手段などは、ホームページ

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旅費を希望する方には補助できる可能性があります。都築 (tsuzuki@math.tohoku.ac.jp) まで早めに連絡して下さい。大学院生・研究生の方は、指導教員経由で申し込み下さい。

プログラム

7月21日(火)

9:30 – 10:30 北山 秀隆 (大阪大学)/Hidetaka Kitayama (Osaka)

On dimension formula for Siegel modular forms with respect to the non-split symplectic groups

10:40 – 11:40 長谷川 泰子 (東京大学)/Yasuko Hasegawa (Tokyo)

Equidistribution of Eisenstein series in various aspects
(joint work with Shinya Koyama)

11:50 – 12:50 河村 尚明 (北海道大学)/Hisa-aki Kawamura (Hokkaido)

On certain p -adic families of cuspidal Siegel modular forms of higher genus

14:00 – 15:00 岡本 卓也 (名古屋大学)/Takuya Okamoto (Nagoya)

Analytic properties of a certain multiple Dirichlet series

15:15 – 16:15 小野寺 一浩 (東京工業大学)/Kazuhiro Onodera (Titech)

Generalized log sine integrals and the Mordell-Tornheim zeta values

16:30 – 17:30 加塩 朋和 (京都大学)/Tomokazu Kashio (Kyoto)

On monomial relations of Barnes's multiple gamma function

7月22日(水)

9:30 – 10:30 吉田 学 (九州大学)/Manabu Yoshida (Kyushu)

Ramification of local fields and Fontaine's property (Pm)

10:40 – 11:40 鈴木 貴士 (京都大学)/Takashi Suzuki (Kyoto)

Some remarks on local class field theory of Serre and Hazewinkel

11:50 – 12:50 大久保 俊 (東京大学)/Shun Ohkubo (Tokyo)

A note on Sen's theory in the imperfect residue field case

14:00 – 15:00 金城 謙作 (東北大学)/Kensaku Kinjo (Tohoku)

2-adic arithmetic-geometric mean and elliptic curves

15:15 – 16:15 杉山 倫 (名古屋大学)/Rin Sugiyama (Nagoya)

On the kernel of the reciprocity map of certain simple normal crossing varieties over finite fields

16:30 – 17:30 橋本 健治 (東京大学)/Kenji Hashimoto (Tokyo)

Period map of a certain family of K3 surfaces over a Shimura curve

7 月 23 日 (木)

9:30 – 10:30 津野 祐司 (中央大学)/Yuji Tsuno (Chuo)

Normal basis problem for torsors under a finite flat group scheme

10:40 – 11:40 小関 祥康 (九州大学)/Yoshiyasu Ozeki (Kyushu)

Eigenvalues of Frobenius and the non-existence of certain Galois representations

11:50 – 12:50 塩見 大輔 (名古屋大学)/Daisuke Shiomi (Nagoya)

On relative congruence zeta functions for cyclotomic function fields

14:00 – 15:00 森澤 貴之 (早稲田大学)/Takayuki Morisawa (Waseda)

Weber's problem in the cyclotomic \mathbb{Z}_3 -extension of the rational field

15:15 – 16:15 伊東 杏希子 (名古屋大学)/Akiko Ito (Nagoya)

On the divisibility of class numbers of imaginary quadratic fields $\mathbb{Q}(\sqrt{2^{2k} - q^n})$ and some Diophantine equations

16:30 – 17:30 三浦 崇 (慶應義塾大学)/Takashi Miura (Keio)

On the ideal class groups of CM-fields

18:00 – 懇親会

7 月 24 日 (金)

9:30 – 10:30 津嶋 貴弘 (東京大学)/Takahiro Tsushima (Tokyo)

Elementary computation of ramified component of the Jacobi sum

10:40 – 11:40 中村 健太郎 (東京大学)/Kentaro Nakamura (Tokyo)

Zariski density of two dimensional trianguline representations of p -adic fields

11:50 – 12:50 Anna Cadoret (Universite de Bordeaux I)

A uniform open image theorem for ℓ -adic representations
(joint work with Akio Tamagawa)

14:00 – 15:00 平之内 俊郎 (京都大学)/Toshiro Hiranouchi (Kyoto)

Pure weight perfect modules on divisorial schemes
(joint work with Satoshi Mochizuki)

15:15 – 16:15 新井 啓介 (東京大学)/Keisuke Arai (Tokyo)

Rational points on $X_0^+(37M)$
(joint work with Fumiyuki Momose)

世話人：松本眞 (広島大学)・都築暢夫 (東北大学)・市原由美子 (広島大学)・西来路文朗 (広島国際大学)・山内卓也 (大阪府立大学)・高橋浩樹 (広島大学)

第 8 回広島整数論集会

アブストラクト

北山 秀隆 大阪大学大学院理学研究科・D3、学振特別研究員 DC2

“On dimension formula for Siegel modular forms with respect to the non-split symplectic groups”

In this talk, we consider the dimension of the spaces of Siegel modular forms of degree two with respect to a certain type of discrete subgroups of the non-split symplectic groups. First, we give an explicit dimension formula for the spaces of weight $k \geq 5$ by using the formula of S.Wakatsuki. This is a generalization of the result of K.Hashimoto. In addition, we consider the spaces of low weights in a certain case by using the method of T.Sugano and Y.Hirai.

長谷川 泰子 東京大学大学院数理科学研究科 D3 (特別研究員 DC2)

“Equidistribution of Eisenstein series in various aspects”

(Joint work with S. Koyama (Ewha Womans Univ, Toyo Univ))

In 1995, Luo and Sarnak proved an equidistribution property of the real analytic Eisenstein series $E(z,s)$ with $\text{Re}(s)=1/2$ as $\text{Im}(s) \rightarrow \infty$. This phenomenon is called quantum ergodicity. In 2008, Koyama proved its analog in the level aspect. More precisely, he proved equidistribution of the Eisenstein series for $\Gamma_0(q)$ with s fixed and with letting $q \rightarrow \infty$. In this talk we introduce generalizations of such phenomenon. Especially, we see that a function field case analog can be considered. It did not exist for the Luo-Sarnak case, because $\text{Im}(s)$ was bounded over function fields. But Koyama’s work enables us to observe an analogous phenomenon. We also introduce our trial towards the Siegel modular case.

河村 尚明 北海道大学大学院理学研究院 学術研究員 (数学部門)

“On certain p -adic families of cuspidal Siegel modular forms of higher genus”

In Recent years, p -adic families of Siegel modular forms of higher genus were considered from various points of view by several people, for example, Tilouine-Urban, Panchishkin and Guerzhoy. In particular, Panchishkin constructed p -adic families of Siegel Eisenstein series, that is, non-cuspidal Siegel modular forms of arbitrary genus. In this talk, we would like to explain some constructions of those of cuspidal Siegel modular forms of higher genus by applying a lifting procedure of cuspidal modular forms of genus one due to Duke-Imamoglu and Ikeda.

岡本 卓也 名大多元・D 1

“Analytic properties of a certain multiple Dirichlet series”

We consider a certain multiple Dirichlet series which is a generalization of that introduced in Masri, and we prove the meromorphic continuation to the whole space. Also, using certain functional relations and the technique of changing variables introduced in Akiyama, Egami and Tanigawa, we prove that “the possible singularities” is indeed “the true singularities”.

小野寺 一浩 東京工業大学・PD

“Generalized log sine integrals and the Mordell-Tornheim zeta values”

We introduce a generalization of Euler’s log sine integrals. It is shown that all the integrals are expressed by the Mordell-Tornheim zeta values at positive integers and the

converse is also true. Moreover, we apply the theory of the integral to obtain various new results for the Mordell-Tornheim zeta values.

加塩 朋和 京都大学大学院理学研究科数学教室・学振特別研究員 (P D)

“On monomial relations of Barnes’s multiple gamma function”

Yoshida’s class invariant $X_F(c)$ is defined by special values of Barnes’s multiple gamma functions. In this talk, we will show some “monomial relations” on his invariants. Additionally, Shintani’s formula expresses derivative values of partial zeta functions in terms of $X_F(c)$. As a result, our “monomial relations” relates to Stark’s conjecture. On the other hand, Yoshida’s conjecture expresses any CM-period, which is the transcendental part of a critical value of L -function associated with an algebraic Hecke character, by the values of $X_F(c)$. Therefore, we may also see the “relation” between CM-periods and Stark’s units.

吉田 学 九州大学大学院数理学府博士課程 1 年 GCOE プログラム TRA

「局所体の分岐と Fontaine の性質 (Pm)」

“Ramification of local fields and Fontaine’s property (Pm)”

K を完全体を剰余体とする完備離散付値体とする。有限次 Galois 拡大 L/K の Galois 群 G に対して、その上付き分岐 filtration (G_j) を考える。このとき、Fontaine は拡大 L/K と実数 m に関するある (Pm) と呼ばれる性質を用いて上付き分岐最大跳躍数を特徴付けた。今回 Fontaine の結果を精密化することで、上付き分岐 filtration の (Pm) による解釈を与える。

Let K be a complete discrete valuation field with perfect residue field. Consider the ramification filtration (G_j) in the upper numbering of the Galois group of a finite Galois extension L of K . Then Fontaine characterized the greatest break of the ramification filtration by a certain property (Pm) of the extension L/K for real numbers m . By refining Fontaine’s result, we obtain a new interpretation of the ramification filtration in terms of the property (Pm).

鈴木 貴士 京都大学大学院理学研究科数学教室博士課程 1 年生

「Serre と Hazewinkel による局所類体論についての注意」

“Some remarks on local class field theory of Serre and Hazewinkel”

Serre と Hazewinkel の局所類体論を簡単に解説した後、それに新しいアプローチを与える。また標数 0 の場合に D -加群版の命題も紹介する。2次元局所類体論もこの枠組みの下で議論する。

We briefly review local class field theory of Serre and Hazewinkel, and give a new approach for this theory. In the case of characteristic zero, we also show a D -module version of the theory. Two-dimensional local class field theory is discussed in this framework.

大久保 俊 東京大学数理学研究科 博士課程 1 年

“A note on Sen’s theory in the imperfect residue field case”

In Sen’s theory in the imperfect residue field case, Brinon defined a functor from the category of \mathbb{C}_p -representations to the category of linear representations of certain Lie algebra. We give a comparison theorem between the continuous Galois cohomology of \mathbb{C}_p -representations and the Lie algebra cohomology of the associated representations. The key ingredients of the proof are Hyodo’s calculation of Galois cohomology and the effaceability of Lie algebra cohomology for solvable Lie algebras.

金城 謙作 東北大学大学院理学研究科数学専攻・(D2)

“2-adic arithmetic-geometric mean and elliptic curves”

It is well-known that there exists a deep relation between the arithmetic-geometric mean and elliptic curves over the real number field. For any prime p , Henniart and Mestre defined the p -adic arithmetic-geometric mean, and related it with the period of an elliptic curve having multiplicative reduction over the p -adic field.

In this talk, we report that the sequence of the ratio of 2-adic arithmetic-geometric mean sequences converges periodically. Moreover, using these convergence, we obtain the canonical lift of the reduction of an elliptic curve having good ordinary reduction over the 2-adic field.

杉山 倫 名古屋大学大学院多元数理科学研究科・博士後期課程 1 年

“On the kernel of the reciprocity map of certain simple normal crossing varieties over finite fields”

The reciprocity map of the unramified class field theory for a proper variety X over a finite field k is a homomorphism of the following form:

$$\rho_X : CH_0(X) \longrightarrow \pi_1^{ab}(X).$$

Here $CH_0(X)$ is the Chow group of 0-cycles on X modulo rational equivalence, and $\pi_1^{ab}(X)$ is the abelian étale fundamental group of X . In this talk, we study the kernel of ρ_X of a certain simple normal crossing variety X over k . We also give an example of a simple normal crossing surface Y for which the map induced by ρ_Y for a positive integer n

$$\rho_{Y \otimes_k F}/n : CH_0(Y \otimes_k F)/n \longrightarrow \pi_1^{ab}(Y \otimes_k F)/n$$

is not injective for any finite extension F/k .

橋本 健治 東京大学大学院数理科学研究科・D2

”Period map of a certain family of K3 surfaces over a Shimura curve”

We study a one-parameter family of K3 surfaces over a Shimura curve, which is a maximal family of algebraic K3 surfaces with an action of alternating group of degree 5. We construct the inverse of the period map of this family using automorphic forms on the period domain.

津野 祐司 中央大学大学院理工学研究科 D 2

“Normal basis problem for torsors under a finite flat group scheme”

We constructed a deformation of the Kummer sequence to the radicial sequence over an F_p -algebra, which is somewhat dual for the deformation of the Artin-Schreier sequence to the radicial sequence, studied by Saidi. There are some relations between our sequences and the embedding of a finite flat commutative group scheme into a connected smooth affine commutative group schemes, constructed by Grothendieck. Furthermore, I will explain that the concept of normal bases in the classical Galois theory is generalized in the framework of group schemes and that the Grothendieck resolution gives a characterization of the torsors with a normal basis.

小関 祥康 九州大学数理学府数理学専攻 博士 2 年・学振特別研究員 DC2

“Eigenvalues of Frobenius and the non-existence of certain Galois representations”

密度 0 の素点の集合の外で不分岐な p -進 Galois 表現で、Frobenius 作用の固有値がある特別な性質を持つようなものが存在しない、ということを紹介する。

In this talk, we prove the non-existence of certain Galois representations unramified outside a density zero set of finite places whose eigenvalues of Frobenius have a special value.

塩見 大輔 名古屋大学多元数理科学研究科、博士後期課程 3 年、学術振興会 特別研究員
“On relative congruence zeta functions for cyclotomic function fields”

In the late 1990s, Rosen. M gave the determinant formula for the relative class number for the P -th cyclotomic function fields in the case of the monic irreducible polynomial P . This result is regarded as an analogue of the classical Maillet determinant. A relative congruence zeta function for a cyclotomic function field can be expressed by a polynomial with coefficients of a integer. In this talk, we will provide the determinant formula for this polynomial. By the analytic class number formula, this formula leads Rosen’s one. As an application of our formula, we will determine low degree terms of this polynomial.

森澤 貴之 早稲田大学大学院基幹理工学研究科数学応用数理専攻 修士課程 2 年
「有理数体の円分的 \mathbb{Z}_3 拡大における Weber の問題」

“Weber’s problem in the cyclotomic \mathbb{Z}_3 -extension of the rational field”

Let h_n be the class number of the n -th layer of the cyclotomic \mathbb{Z}_p -extension over the rational field. Washington proved that the ℓ -part of h_n is bounded as n tends to ∞ for a fixed prime number ℓ . So we are interested in the triviality of the ℓ -part of h_n for all positive integer n . In this talk, we talk about the case $p = 3$.

伊東 杏希子 名古屋大学大学院多元数理科学研究科 博士後期過程 1 年
「虚二次体 $\mathbb{Q}(\sqrt{2^{2k} - q^n})$ の類数の可除性と関連するディオファントス方程式について」
“On the divisibility of class numbers of imaginary quadratic fields $\mathbb{Q}(\sqrt{2^{2k} - q^n})$ and some Diophantine equations”

In this talk, we show that class numbers of imaginary quadratic fields $\mathbb{Q}(\sqrt{2^{2k} - q^n})$ are divisible by n , where q is an odd prime number, n is a positive integer and $k \geq 1$ are integers with $2^{2k} < q^n$. This is a generalization of the result of Kishi for imaginary quadratic fields $\mathbb{Q}(\sqrt{2^{2k} - 3^n})$. We will also discuss some Diophantine equations concerning the divisibility of class numbers of quadratic fields.

三浦 崇 慶応義塾大学理工学研究科 博士 1 年
“On the ideal class groups of CM-fields”

Let k be a totally real number field and F be a CM-field which is abelian over k . It is important to know the action of $\text{Gal}(F/k)$ on the ideal class group of F . Brumer’s conjecture is a generalization of Stickelberger’s theorem, and gives some information on the Galois action. Brumer’s conjecture was almost proved in the case that the p -adic L functions have no trivial zero, but is still open in general. In this talk, we will prove this conjecture assuming a certain strong condition, but in our case the p -adic L functions may have trivial zeros. Moreover, we will discuss the stronger version of Brumer’s conjecture and study the Fitting ideal of $Cl(F)$. This is a joint work with Masato Kurihara.

津嶋 貴弘 東京大学数理科学研究科 博士 3 年・学術振興会特別研究員 (DC1)
「Jacobi 和の量指標の局所成分の初等的な計算」

“Elementary computation of ramified component of the Jacobi sum”

R. Coleman and W. McCallum calculated the Jacobi sum Hecke characters explicitly using their computation of the stable reduction of the Fermat curve in 1988. We give an

elementary proof of the main result of them without using rigid geometry or depending on a full understanding of the stable model of the Fermat curve.

中村 健太郎 東京大学大学院数理科学研究科 研究生

「 p 進体の二次元三角表現の Zariski 稠密性」

“Zariski density of two dimensional trianguline representations of p -adic fields”

三角表現 (trianguline 表現) とは、 p 進体の p 進ガロア表現のクラスであり、近年 p 進局所ラングランズ対応などとの関連から脚光を浴びるようになってきている。本講演では、一般の p 進体に対して、二次元三角表現が p 進体の二次元 p 進ガロア表現の変形空間の中で Zariski 稠密に含まれている、という内容の定理を証明する。なお、 p 進体が Q_p の場合は、この定理は Colmez, Kisin により既に証明されており、本講演の主定理は彼らの結果の、一般の p 進体の場合への一般化にあたる。

Trianguline representation is a class of p -adic Galois representations of a p -adic field. This is the most important class in the study of p -adic local Langlands correspondence. In my talk, for any p -adic fields, we prove Zariski density of two dimensional trianguline representations in deformation spaces of p -adic Galois representations. When p -adic field is Q_p , this theorem was proved by Colmez and Kisin, our theorem in my talk is the generalization of their theorem for any p -adic fields case.

Anna Cadoret Université de Bordeaux I

“A uniform open image theorem for ℓ -adic representations”

(joint work with Akio Tamagawa - RIMS)

Let k be a finitely generated field of characteristic 0, X a smooth, separated, geometrically connected curve over k with generic point η . A ℓ -adic representation $\rho : \pi_1(X) \rightarrow \mathrm{GL}_m(\mathbb{Z}_\ell)$ is said to be geometrically strictly nonabelian (GSNA for short) if $\mathrm{Lie}(\rho(\pi_1(X_{\bar{k}})))^{ab} = 0$. Typical examples of such representations are those arising from the action of $\pi_1(X)$ on the generic ℓ -adic Tate module $T_\ell(A_\eta)$ of an abelian scheme A over X or, more generally, from the action of $\pi_1(X)$ on the ℓ -adic étale cohomology groups $H^i(Y_{\bar{\eta}}, \mathbb{Q}_\ell)$, $i \geq 0$ of the geometric generic fiber of a smooth proper scheme Y over X . Let G denote the image of ρ . Any closed point x on X induces a splitting $x : \Gamma_{\kappa(x)} := \pi_1(\mathrm{Spec}(\kappa(x))) \rightarrow \pi_1(X_{\kappa(x)})$ of the canonical restriction epimorphism $\pi_1(X_{\kappa(x)}) \rightarrow \Gamma_{\kappa(x)}$ (here, $\kappa(x)$ denotes the field of definition of x) so one can define the closed subgroup $G_x := \rho \circ x(\Gamma_{\kappa(x)}) \subset G$. The main result I am going to discuss is the following uniform open image theorem (and its application to uniform boundedness of the ℓ -primary torsion on abelian varieties). *Under the above assumptions, for any GSNA representation $\rho : \pi_1(X) \rightarrow \mathrm{GL}_m(\mathbb{Z}_\ell)$ and any integer $d \geq 1$, the set $X_{\rho,d}$ of all closed points $x \in X$ such that G_x is not open in G and $[\kappa(x) : k] \leq d$ is finite and there exists an integer $B_{\rho,d} \geq 1$ such that $[G : G_x] \leq B_{\rho,d}$ for any closed point $x \in X \setminus X_{\rho,d}$ such that $[\kappa(x) : k] \leq d$.*

The statement for arbitrary d is a consequence of the statement for $d = 1$. In this talk, I will assume the case $d = 1$ and focus on how to deduce from it the general statement.

A key ingredient of our proof is that, for any integer $\gamma \geq 1$ there exist an integer $\nu = \nu(\gamma) \geq 1$ such that, given any projective system $\cdots \rightarrow Y_{n+1} \rightarrow Y_n \rightarrow \cdots \rightarrow Y_0$ of curves (over an algebraically closed field of characteristic 0) with the same gonality γ and with $Y_{n+1} \rightarrow Y_n$ a G -cover of degree > 1 , one can construct a projective system of genus 0 curves $\cdots \rightarrow B_{n+1} \rightarrow B_n \rightarrow \cdots \rightarrow B_\nu$ and degree γ morphisms $f_n : Y_n \rightarrow B_n$, $n \geq \nu$ such that Y_{n+1} is birational to $B_{n+1} \times_{B_n, f_n} Y_n$, $n \geq \nu$.

REFERENCES

- [CT08a] A. CADORET and A. TAMAGAWA, *Uniform boundedness of p -primary torsion of abelian schemes*, preprint, 2008.
- [CT08b] A. CADORET and A. TAMAGAWA, *Torsion of abelian schemes and rational points on moduli spaces*, preprint, 2008, to appear in RIMS Kokyuroku Bessatsu.
- [CT08c] A. CADORET and A. TAMAGAWA, *A uniform open image theorem for ℓ -adic representations I*, preprint, 2009.
- [CT08d] A. CADORET and A. TAMAGAWA, *A uniform open image theorem for ℓ -adic representations II*, preprint, 2009.

All references available at: <http://www.math.u-bordeaux.fr/~cadoret/>.

平之内 俊郎 京都大学数理解析研究所・特定研究員 (GCOE)

“Pure weight perfect Modules on divisorial schemes”

Gersten’s conjecture (1974) on localization sequences of Quillen K-theory is: For a regular closed immersion $Z \rightarrow X$ of codimension r on a certain scheme X , the homotopy fiber in K-theory of the restriction map $K(X) \rightarrow K(X - Z)$ is given by the K-theory of an exact category. This generalizes a well-known result of Quillen (1976) who proved the case $r=1$ and $X-Z$ affine. We extend this conjecture to non-connected (resp. additive) K-theory and prove it. We also refer to an application to Weibel’s conjecture on negative K-groups.

新井 啓介 東京大学大学院 数理科学研究科 研究生

東京電機大学 工学部・未来科学部 非常勤講師

「 $X_0^+(37M)$ の有理点」(百瀬文之氏との共同研究)

“Rational points on $X_0^+(37M)$ ” (joint work with Fumiyuki Momose)

Momose studied rational points on the modular curve $X_0^+(N)$ for a composite number N which has a prime divisor p different from 37. The prime $p = 37$ is peculiar because $X_0(37)$ is a hyperelliptic curve and w_{37} is not the hyperelliptic involution. We show that the rational points on $X_0^+(37M)$ consist of cusps and CM points. We also generalize the result for imaginary quadratic fields.

百瀬氏は、37 とは異なる素数 p で割れるような合成数 N に対して、モジュラー曲線 $X_0^+(N)$ の有理点を調べた。素数 $p = 37$ は、 $X_0(37)$ が hyperelliptic curve であるのに w_{37} は hyperelliptic involution ではないという点で、特殊である。ここでは、 $X_0^+(37M)$ の有理点がカスプと CM 点のみから成ることを示す。また、その結果を虚二次体の場合にも拡張する。
