# 第 9 回仙台広島整数論集会

下記の日程で研究集会を開催します。

2010年7月20日(火)-7月23日(金)

東北大学大学院理学研究科 (青葉山キャンパス) 数理科学記念館 (川井ホール)

本研究集会は、科研費基盤研究 (A)「数論・幾何の新展開:数論的トポロジー、圏論的数論幾何、アルゴリズム」(代表者 松本眞) および科研費基盤研究 (B)「p 進的手法を用いた数論的多様体の研究」(代表者 都築暢夫) からの補助を受けております。なお、2010 年 3 月終了の日本学術振興会 先端研究拠点事業 (No.18005) 「数論幾何・モチーフ理論・ガロア理論の新展開と、その社会的実用」(コーディネーター 松本眞)のセミナーのシリーズです。

プログラムの変更・講演のアブストラクト・会場までの交通手段などは、ホームページ

http://www.math.sci.hiroshima-u.ac.jp/~m-mat/JSPS-CoreToCore/SEISURON10/hiroshima10.html

をご覧下さい。

旅費を希望する方には補助できる可能性があります。都築 (tsuzuki@math.tohoku.ac.jp) まで早めに連絡して下さい。大学院生・研究生の方は、指導教員経由で申し込み下さい。

#### プログラム

7月20日(火)

9:30 — 10:30 岡本 卓也 (名古屋大学)/Takuya Okamoto (Nagoya) Partial Mordell-Tornheim multiple zeta values

10:40 – 11:40 佐々木 義卓 (近畿大学)/Yoshitaka Sasaki (Kinki)
On generalized poly-Bernoulli numbers and related *L*-functions

11:50 – 12:50 大下 達也 (京都大学)/Tatsuya Ohshita (Kyoto)
The Euler systems of cyclotomic units and higher Fitting ideals

14:00 – 15:00 大槻 玲 (慶應義塾大学)/Rei Otsuki (Keio)

Construction of p-adic Hecke L-functions using the Kronecker theta function in the cyclotomic supersingular case

15:15 – 16:15 吉田 学 (九州大学)/Manabu Yoshida (Kyushu)

An ultrametric space of Eisenstein polynomials and ramification theory

16:30 — 17:30 平之内 俊郎 (広島大学)/Toshiro Hiranouchi (Hiroshima) On a filtration associated with an isogeny on formal groups

7月21日(水)

9:30 – 10:30 竹森 翔 (京都大学)/Sho Takemori (Kyoto) A p-adic limit of Siegel-Eisenstein series of degree two

10:40-11:40 宗野 惠樹 (東京大学)/Keiju Sono (Tokyo)

The matrix coefficients with minimal K-types of the spherical and non-spherical principal series representations of  $SL(3,\mathbb{R})$ 

- 11:50 12:50 山名 俊介 (大阪市立大学)/Shunsuke Yamana (Osaka City) The Siegel-Weil formula for quaternionic unitary groups
- 14:00 15:00 高井 勇輝 (名古屋大学)/Yuuki Takai (Nagoya)
  An effective isomorphy criterion for mod ℓ Galois representations
- 15:15 16:15 橋本 康史 (九州先端科学技術研究所)/Yasufumi Hashimoto (ISIT)
  Asymptotic formulas of class number sums of indefinite binary quadratic forms
- 16:30 17:30 許斐 豊 (学習院大学)/Yutaka Konomi (Gakushuin)

  The ideal class groups of dihedral extensions over imaginary quadratic fields and the special values of the Artin *L*-functions

## 7月22日(木)

- 9:30 10:30 **宮谷 和尭** (東京大学)/Kazuaki Miyatani (Tokyo) Finiteness of Crystalline Cohomology of Higher Level
- 10:40 11:40 中川 貴裕 (千葉大学)/Takahiro Nakagawa (Chiba) Calculation of p-adic unbounded functions on the unit disc
- 11:50 12:50 中村 健太郎 (慶應義塾大学)/Kentaro Nakamura (Keio) Zariski density of crystalline representations
- 14:00 15:00 津嶋 貴弘 (東京大学)/Takahiro Tsushima (Tokyo) On the stable reduction of the modular curve  $X_0(p^4)$
- 15:15 16:15 三枝 洋一 (九州大学)/Yoichi Mieda (Kyushu) Variants of formal nearby cycles and applications
- 16:30 17:30 望月 哲史 (東洋大学)/Satoshi Mochizuki (Toyo)
  Weight, unstable dimension, some conjectures in algebraic geometry via noncommutative motive theory
- 18:00 懇親会 理薬厚生施設「グリーンホール」内レストラン AOSIS

#### 7月23日(金)

- 9:30 10:30 小島 彰太 (立教大学)/Shota Kojima (Rikkyo) Infinite Composition of Polynomials sum
- 10:40 11:40 金子 元 (京都大学)/Hajime Kaneko (Kyoto) Algebraic independence of the values of power series
- 11:50 12:50 岡本 亮彦 (早稲田大学)/Akihiko Okamoto (Waseda) Relative Brauer groups and tractability of algebraic function fields in one variable
- 14:00 15:00 塩見 大輔 (名古屋大学)/Daisuke Shiomi (Nagoya) A p-rank of Jacobian for a cyclotomic function field
- 15:15 16:15 内田 幸寛 (京都大学)/Yukihiro Uchida (Kyoto)
  Valuations of Somos 4 sequences and local heights on elliptic curves

世話人: 平之内 俊郎 (広島大学)・松本 眞 (東京大学)・高橋 浩樹 (広島大学)・都築 暢夫 (東北大学)・ 雪江 明彦 (東北大学)

# 第 9 回仙台広島整数論集会

# アブストラクト

# 岡本 卓也 (Takuya Okamoto) 名古屋大学多元数理科学研究科 D2 部分 Mordell-Tornheim 多重ゼータ値

Partial Mordell-Tornheim multiple zeta values

We define the partial Mordell-Tornheim multiple zeta functions by

$$\zeta_{MT,r}(s_1,\ldots,s_r;s_{r+1}) = \sum_{m_1,\ldots,m_r}^{\infty} \frac{1}{m_1^{s_1}\cdots m_r^{s_r}(2m_1+m_2+\cdots+m_r)_{r+1}^s}$$

for  $r \in \mathbb{N}$  and  $s_1, \ldots, s_r \in \mathbb{C}$ , and consider the values of the partial Mordell-Tornheim multiple zeta function  $\zeta_{MT,r}$  at positive integers by using the basic properties of Milnor's multiple sine function and Bernoulli polynomials which is introduced by Onodera.

# 佐々木 義卓 (Yoshitaka Sasaki) 近畿大学・理工学部 (博士研究員)PD

#### 一般 Bernoulli 数の poly 化と付随する L 関数の構成について

On generalized poly-Bernoulli numbers and related L-functions

Kaneko introduced poly-Bernoulli numbers which are generalization of Bernoulli numbers. After that Arakawa and Kaneko introduced a new zeta-function (Arakawa-Kaneko's zeta-function) whose special values at non-positive integers are expressed by poly-Bernoulli numbers. I plan to talk about the generalization of the Dirichlet *L*-functions based on Arakawa-Kaneko's zeta-function. Further I introduce generalized poly-Bernoulli numbers via special values at non-positive integers of such *L*-functions.

# 大下 達也 (Tatsuya Ohshita) 京都大学理学研究科数学教室 (博士後期課程 1 回生) 円単数の Euler 系と高次 Fitting イデアル

The Euler systems of cyclotomic units and higher Fitting ideals

Masato Kurihara (Keio University) established a refinement of the minus-part of the Iwasawa main conjecture of ideal class groups. In this talk, we will explain a result on the higher Fitting ideals of the plus-part. Using the Euler system of cyclotomic units, we will construct ideals of the Iwasawa algebra which give upper bounds of the higher Fitting ideals. This result can be regarded as a refinement of the plus-part of the Iwasawa main conjecture.

#### 大槻 玲 (Rei Otsuki) 慶應義塾大学理工学部数理科学科 特別研究助教

Construction of p-adic Hecke L-functions using the Kronecker theta function in the cyclotomic supersingular case

In this talk, following an idea of Shinichi Kobayashi, we give a construction of cyclotomic p-adic L-functions in the supersingular and general weight case which interpolate special values of Hecke L-functions associated to CM elliptic curves by using the Kronecker theta function. These p-adic L-functions are essentially a special case of the p-adic L-functions obtained by Vishik and Amice-Vélu.

#### 吉田 学 (Manabu Yoshida) 九州大学大学院数理学府 (博士後期課程二年)

An ultrametric space of Eisenstein polynomials and ramification theory

We give a criterion whether two given polynomials over a local field define the same extension in terms of a certain non-Archimedean metric on the set of polynomials. The criterion and its proof depend on ramification theory. We introduce some applications for Galois representations and truncated discrete valuation rings. (joint work with T. Suzuki)

## 平之内 俊郎 (Toshiro Hiranouchi) 広島大学理学研究科数学専攻 (助教)

On a filtration associated with an isogeny on formal groups

We study a filtration associated with an isogeny on formal groups over a local field. As an application, the image of the Kummer map on supersingular elliptic curves is decided. For the product of supersingular elliptic curves, the image of the Albanese kernel by the cycle map is also calculated. (joint work with Seiji Hirayama)

### 竹森 翔 (Sho Takemori) 京都大学大学院理学研究科 D1

A p-adic limit of Siegel-Eisenstein series of degree two

We introduce an explicit formula for Fourier coefficients of Siegel-Eisenstein series of degree two for all level, and consider a p-adic limit of Fourier coefficients of Siegel-Eisenstein series and a p-adic analytic family of Siegel-Eisenstein series of degree two. If weight is larger than 3, this results is a generalization of Katsurada-Nagaoka's work and Mizuno's work.

### 宗野 惠樹 (Keiju Sono) 東京大学大学院数理科学研究科 博士課程 2 年

The matrix coefficients with minimal K-types of the spherical and non-spherical principal series representations of  $SL(3,\mathbb{R})$ 

We compute the holonomic system of rank 6 for the radial part of the matrix coefficients of class one and non-spherical principal series of  $SL(3,\mathbb{R})$ . We give explicit formulas of the coefficients of six power series solutions, and express the matrix coefficients by their linear combinations. Among others, the c- functions of non-spherical principal series are obtained.

## 山名 俊介 (Shunsuke Yamana) 大阪市立大学・大学院理学研究科・特別研究員 (PD) 四元数ユニタリ群のジーゲル・ヴェイユ公式

The Siegel-Weil formula for quaternionic unitary groups

The Siegel-Weil formula is an identity between a value of a certain Eisenstein series and an integral of a theta function. Such identity was first proven by Siegel and then extended to classical dual reductive pairs by Weil under the assumption that the Eisenstein series is absolutely convergent. Later, Kudla and Rallis extended this formula for symplectic-orthogonal dual pairs beyond the range of absolute convergence. In this talk we extend the Siegel-Weil formula for quaternion dual pairs. If there is time, applications include the classification problem of quaternion skew hermitian forms over a number field.

### 高井 勇輝 (Yuuki Takai) 名古屋大学多元数理 (特任助教)

An effective isomorphy criterion for mod  $\ell$  Galois representations

In this talk, we consider mod  $\ell$  Galois representations of  $\mathbb{Q}$ . In particular, we obtain an effective criterion to distinguish two absolutely irreducible 2-dimensional, odd, mod  $\ell$  Galois representations up to isomorphism. Serre's conjecture (now, Khare-Wintenberger's theorem), Sturm's theorem for mod  $\ell$  modular forms, and its modification by Kohnen are used in our proof.

# 橋本 康史 (Yasufumi Hashimoto) 九州先端科学技術研究所 (研究員) 不定値二元二次形式の類数和に関する漸近公式

Asymptotic formulas of class number sums of indefinite binary quadratic forms

It is known that there is a one-to-one correspondence between equivalence classes of primitive indefinite quadratic forms and primitive hyperbolic conjugacy classes of the modular group. This means that, as studied by Sarnak, the asymptotic formula for the class number sum (which is different to Siegel's one) follows from the prime geodesic theorem for the modular group. Recently, Raulf gave asymptotic formulas for the class number sums over the fundamental discriminants on arithmetic progressions. In the present paper, we write down the coefficient of the leading terms of such asymptotic formulas in another way and improve the estimates of the reminder terms.

# 許斐 豊 (Yutaka Konomi) 学習院大学理学部 (客員研究員) 虚二次体上の二面体拡大のイデアル類群とアルティン L 関数の特殊値

The ideal class groups of dihedral extensions over imaginary quadratic fields and the special values of the Artin L-functions

Let L be a dihedral extension over the rational number field of degree 2l, where l is an odd prime. Denote by G its Galois group. For another odd prime p, we investigate the relationship between the p-class subgroups defined by the characters of subgroup of G and those defined by the irreducible characters. Moreover, we try to describe, by the special value of the Artin L-function, the order of the p-class subgroup of a dihedral extension over an imaginary quadratic field.

# 宮谷 和尭 (Kazuaki Miyatani) 東京大学大学院数理科学研究科博士課程 1 年

Finiteness of Crystalline Cohomology of Higher Level

The main object of this talk, the crystalline cohomology of higher level, is a generalization of the classical crystalline cohomology to "ramified base DVRs," which was introduced by Berthelot around 1990. The integral theory of this cohomology is difficult to treat because the "crystalline Poincare lemma" is very complicated in the higher-level situation. In spite of this complexity, we see in this talk how to prove the finiteness of this cohomology and other fundamental properties.

#### 中川 貴裕 (Takahiro Nakagawa) 千葉大学院理学研究科 D 2

Calculation of p-adic unbounded functions on the unit disc

Let  $f = \sum_{n \geq 0} a_n x^n$  be the power series which converge on the open unit disc over p-adic local field. The coefficients of f may be unbounded.  $(\Leftrightarrow |a_n| \to \infty (n \to \infty))$  Log-growth of f is defined the order of unboundness of f.

Since relation between log-growth and Frobenius filtrations for solutions of p-adic differential equations is open problem, to study structure of log-growth may be important.

By the way, Boutabaa and Escassut composed p-adic Nevanlinna theory on the meromorphic functions over open disc. And they proof that f is bounded function if and only if characteristic function of f is bounded.

This time, by generalizing the theorem, it found that correspondence between log-growth of f and growth of characteristic function of f. From this, log-growth of f is caluculated critical radius and of number of zeros of f.

#### 中村 健太郎 (Kentaro Nakamura) 慶應数理 助教

Zariski density of crystalline representations

In this talk, I will talk about one proof of the theorem concerning Zarisiki denisty of n-dimensional crystalline representations for any positive integer n and for any p-adic field. This result is a generalization of the result of Colmez and Kisin (n=2 and  $\mathbb{Q}_p$  case) and the result which I talked at this conference last year (n=2 and any p-adic fields case). The main two ingredients of the proof are the deformation theory of trianguline B-pairs and the construction of the p-adic families of trianguline representations. In this talk, I want to talk especially about the p-adic families of trianguline representations. Moreover, I will talk about a recent work of Gaetan Chenevier concerning a new construction of the p-adic families of trianguline representations.

# 津嶋 貴弘 (Takahiro Tsushima) 東京大学数理科学研究科・GCOE 特任研究員 On the stable reduction of the modular curve $X_0(p^4)$

By the Deligne-Mumford theorem, a curve C over a local field K has a unique stable model over some finite extension K'/K, as long as the genus of C is greater than one. If defining equations of a curve are given, it is not always easy to compute the stable model of the curve concretely. In our talk, we concentrate on the stable model of a modular curve. The stable models of  $X_0(p), X_1(p)$  and X(p) are classically known by works of Igusa and Deligne-Rapoport. B. Edixhoven calculated the stable model of  $X_0(p^2)$  in 1990. In 2006, Coleman-McMurdy found the stable model of  $X_0(p^3)$  on the basis of the rigid geometry. They use the Gross-Hopkins theory to calculate irreducible components appearing in the stable reduction of  $X_0(p^3)$ . We will introduce a more elementary method to calculate the defining equations of the irreducible components in the model of  $X_0(p^3)$ . Starting with Kronecker's polynomial, we will find components by only using blow-up. By this method, we also find the stable reduction of  $X_0(p^4)$ . Deligne-Lusztig curves for  $SL_2(\mathbb{F}_p)$  appear in the stable model of  $X_0(p^4)$ . In our talk, we will explain how to compute the irreducible components appearing in this model of  $X_0(p^4)$ .

# 三枝 洋一 (Yoichi Mieda) 九州大学大学院数理学研究院 助教 形式隣接輪体の変種とその応用

Variants of formal nearby cycles and applications

The formal nearby cycle for a formal scheme over a complete discrete valuation ring, due to Berkovich, plays a very important role for studies of Shimura varieties and Rapoport-Zink spaces. In this talk, I will introduce variants of the formal nearby cycle and explain its applications to a study of the l-adic cohomology of Rapoport-Zink spaces.

### 望月 哲史 (Satoshi Mochizuki) 東洋大学非常勤講師

Weight, unstable dimension, some conjectures in algebraic geometry via non-commutative motive theory

In the lecture, we consider motive theory as a grammer of how to behave algebraic varieties. More precisely, like as (non)-Euclidean geometry, we think the definition of the usual algebraic varieties as one of a model to do algebraic geometry and propose another model, so called, derived algebraic varieties which include the classical schemes and DG-categories. In this lecture, a non-commutative motive theory, we mean, is studying derived algebraic varieties up to derived Morita or K-equivalences. We recover famous formulas and techniques of algebraic geometry and algebraic K-theory via the non-commutative motive theory setting. For example, localization, excision, blow-up formula, projective

bundle theorem, Balmer formula,  $\mathbb{A}^1$ -homotopy invariance for regular noetherian classical schemes, derived Gillet-Waldhausen theorem, Eilenberg swindle. We also define notions of weight and unstable dimension for (derived) algebraic varieties. Utilizing these notions, we reinterpret some conjectures in algebraic geometry and give partial affermative answers.

# 小島 彰太 (Shota Kojima) 立教大学大学院理学研究科博士課程数学専攻 2 年 Infinite Composition of Polynomials

The exponential function  $e^z$  can be expressed by infinite composition of polynomials of degree 2. Generalizing this special case, we consider functions defined by infinite composition of polynomials and discuss their analytic properties.

The exponential function  $e^z$  can be expressed by infinite composition of polynomials  $z + z^2/2^n$   $(n = 1, 2, 3, \cdots)$ . Generalizing this special case, we consider functions defined by infinite composition of polynomials and discuss their analytic properties.

## 金子 元 (Hajime Kaneko) 京都大学理学研究科 (特別研究員 (PD))

Algebraic independence of the values of power series

We introduce algebraic independence of the numbers  $\xi = \sum_{n=1}^{\infty} \alpha^{-w(n)}$ , where  $\alpha \geq 2$  is an integer and  $(w(n))_{n=1}^{\infty}$  is a strictly increasing sequence of nonnegative integers. In this talk we will give new criteria for algebraic independence of such  $\xi$ . Applying our criteria, we prove the algebraic independence of  $\xi$  in the case of  $\lim_{n\to\infty} w(n+1)/w(n) = 1$ , which was impossible by early methods.

### 岡本 亮彦 (Akihiko Okamoto) 早稲田大学基幹理工学研究科博士課程 4 年

Relative Brauer groups and tractability of algebraic function fields in one variable

A field K is said to be tractable when a condition described below on the simultaneous representation of quaternion algebras holds over K. In this talk, we describe the relative Brauer groups of algebraic function fields in one variable over global fields. As an application, we give examples of tractable or nontractable function fields of genus one.

#### 塩見 大輔 (Daisuke Shiomi) 名古屋大学多元数理科学研究科 PD

A p-rank of Jacobian for a cyclotomic function field

A cyclotomic function field is defined as adding torsion points of Carlitz module to a rational function field over a finite field with characteristic p. Properties of these fields are remarkably similar to those of cyclotomic number fields. In this talk, we will state some results of a p-rank  $r_m$  of Jacobian for a cyclotomic function field  $K_m$ . In paticular, we will determine all cyclotomic function fields  $K_m$  with  $r_m = 0$ .

# 内田 幸寛 (Yukihiro Uchida) 京都大学大学院理学研究科日本学術振興会特別研究員 (PD) Somos 4 数列の付値と楕円曲線上の局所高さ

Valuations of Somos 4 sequences and local heights on elliptic curves

For an integer  $k \geq 4$ , a Somos k sequence is a sequence of numbers defined by a certain bilinear recurrence relation of order k. For example, an elliptic divisibility sequence, which consists of values of division polynomials of an elliptic curve, is a Somos 4 sequence. In this talk, we first describe valuations of Somos 4 sequences in terms of local heights on elliptic curves, where we deal with both Archimedean and non-Archimedean valuations. Next, using this description, we study the asymptotic behavior of valuations of Somos 4 sequences.