

## 第 15 回仙台広島整数論集会アブストラクト

期間 : 2016 年 7 月 12 日 (火) ~ 7 月 15 日 (金)

会場 : 東北大学大学院理学研究科 川井ホール (数理科学記念館)

戸次 鵬人/Hohto Bekki (東京大学/University of Tokyo)

On periodicity of geodesic continued fractions

In this talk, we present geodesic generalizations of Lagrange's theorem in the classical theory of continued fractions. As a result, we establish multi-dimensional continued fraction algorithms to "expand" bases of number fields with rank one unit groups, and prove their periodicity. Furthermore, we show that their periods describe the unit groups of the number fields. Our arguments are adélic so that we can also treat  $p$ -units. We also treat relative quadratic extensions of number fields with rank one relative unit groups.

巖冬/Dong Yan (大阪大学/Osaka University)

モジュラー形式に付随した  $p$  進ガロワ表現の安定格子と肥田変形

Stable lattices in modular Galois representation and Hida deformation

For a  $p$ -adic Galois representation  $V$ , an isomorphic class of Galois stable lattices is not unique when the residual representation of  $V$  is reducible. Also, the number of isomorphic class of Galois stable lattices is finite if  $V$  is semi-simple. In this talk, I first describe the number of isomorphic classes of lattices in terms of some invariants associated to a given modular  $p$ -adic Galois representation. Then I will formulate the problem and I will give some results on the variation of the number of isomorphic class of lattices in Hida deformation when the weight  $k$  varies in the deformation.

平之内 俊郎/Toshiro Hiranouchi (広島大学/Hiroshima University)

A Hermite-Minkowski type theorem of varieties over finite fields

We show the finiteness of étale coverings of a variety over a finite field with given degree whose ramification bounded along an effective Cartier divisor. The proof is an application of P. Deligne's theorem (H. Esnault and M. Kerz in Acta Math. Vietnam. 37:531–562, 2012) on a finiteness of  $l$ -adic sheaves with restricted ramification. By applying our result to a smooth curve over a finite field, one obtain a function field analogue of the classical Hermite-Minkowski theorem.

伊東 良純/Ryojun Ito (千葉大学/Chiba University)

The Beilinson Conjecture for CM Elliptic Curves via Hypergeometric Functions

We consider certain CM elliptic curves which are related to Fermat curves, and express the values of  $L$ -functions at  $s = 2$  in terms of special values of generalized hypergeometric functions. We compare them and a similar result of Rogers-Zudilin with Otsubo's regulator formulas, and give a new proof of the Beilinson conjecture originally due to Bloch.

片岡 武典/Takenori Kataoka (東京大学/University of Tokyo)

A consequence of Greenberg's generalized conjecture on Iwasawa invariants of  $\mathbb{Z}_p$ -extensions

Greenberg's generalized conjecture claims the pseudo-nullity of the unramified Iwasawa module of the maximal multiple  $\mathbb{Z}_p$ -extension of a number field  $k$ . When  $k$  is an imaginary quadratic field, it is known that GGC implies that the Iwasawa invariants associated to many  $\mathbb{Z}_p$ -extensions of  $k$  are small in some sense. In this talk, we partially generalize it to arbitrary number fields  $k$ .

郡田 亨/Tohru Kohrita (名古屋大学/Nagoya University)

### コンパクト台付きモチビック・コホモロジーの代数的部分

Algebraic part of motivic cohomology with compact supports

As a version of algebraic part of Chow groups, we define the algebraic part of motivic cohomology with compact supports for arbitrary smooth varieties over an algebraically closed field. We study these algebraic parts in relation to the universal regular homomorphisms with targets in the category of semi-abelian varieties.

町出 智也/Tomoya Machine (国立情報学研究所/National Institute of Informatics)

### 二つのタイプの正規化多重ゼータ値を含む対称的公式

Symmetric identities involving two types of regularized multiple zeta values

There are two types of regularized multiple zeta values: harmonic and shuffle types. In the paper (2015, Kyushu J. Math. 69, 345–366), Hoffman proved a symmetric identity for harmonic type. In this talk, we give a symmetric identity for shuffle type. In the proof, we use the combinatorial interpretation of coefficients in the Taylor series of the Gamma function, which is stated in the book of Stanley (Algebraic combinatorics. Walks, trees, tableaux, and more).

横山 賢治/Kenji Makiyama (京都産業大学/Kyoto Sangyo University)

A  $p$ -adic analytic family of the  $D$ -th Shintani lifting for a Coleman family and congruences between the central  $L$ -values.

We will construct a  $p$ -adic analytic family of Kojima and Tokuno's  $D$ -th Shintani lifting for a Coleman family and have a  $p$ -adic  $L$ -function which interpolates the central  $L$ -values attached to a Coleman family. Focusing on the case of  $p$ -ordinary, we will obtain two applications. One of them states how a congruence between Hecke eigenforms of different weights derives a congruence between their central  $L$ -values. The other one is about the Goldfeld conjecture in analytic number theory.

松坂 俊輝/Toshiki Matsusaka (九州大学/Kyushu University)

Some formulas for the Fourier coefficients of Hauptmoduln of higher level.

Kaneko gave arithmetic formulas for the Fourier coefficients of the  $j$ -invariant. As analogues, we obtain arithmetic formulas for the coefficients of Hauptmoduln with respect to  $\Gamma_0(p)$  and Fricke group  $\Gamma_0^*(p)$  for  $p = 2, 3$ , and  $5$ . We also obtain their asymptotic formulas by employing Murty-Sampanth's method without the circle method.

三原 朋樹/Tomoki Mihara (東京工業大学/Tokyo Institute of Technology)

### 局所体上の GNS 構成と Krein-Milman の定理について

On GNS construction and Krein-Milman theorem over a local field

GNS construction is a classical method to construct a unitary representation of a group over  $\mathbb{C}$  by using a functional called a state. The notion of a state also appears in the formulation of a physical state in quantum dynamics in physics. The notions of a pure state and a mixed state in quantum dynamics are also formulated in terms of functionals. A pure state precisely corresponds to an extreme point of the convex set of states, and Krein–Milman theorem ensures that the convex set of states is given as the closure of the subset of mixed states. In my talk, I will introduce non-Archimedean analogues of GNS construction and Krein–Milman theorem, and explain a correspondence of the block decompositions of a cyclic semisimple representation of a group and a mixed state through the non-Archimedean GNS construction under the assumption that the coefficient field is of characteristic 0.

宮之原 永士/Eiji Miyanohara (早稲田大学/Waseda University)

A generalization of Erdős-Surányi problem

Erdős-Surányi and Prielipp suggested to study the following problem: For any integers  $k > 0$  and  $n$ , are there an integer  $N$  and a map  $\epsilon : \{1, \dots, N\} \rightarrow \{-1, 1\}$  such that

$$n = \sum_{j=1}^N \epsilon(j)j^k ? \quad (1)$$

Mitek and Bleicher independently solved this problem affirmatively.

In this talk we consider the case that for some positive odd integer  $L$  the numbers  $\epsilon(j)$  are  $L$ -th roots of unity. We show that the answer to the corresponding question is negative if and only if  $L$  is a prime power.

宮崎 弘安/Hiroyasu Miyazaki (東京大学/University of Tokyo)

モデュラス付き高次チャウ群のキューブ不変性

Cube-invariance of higher Chow groups with modulus

The higher Chow group with modulus of a pair  $(X, D)$ , where  $X$  is an algebraic scheme and  $D$  a Cartier divisor, is an object introduced by Binda-Saito as a common generalization of Bloch's higher Chow group and the additive higher Chow group. It is expected to provide the relative cohomology theory. Taking a pair  $(X, \emptyset)$ , it coincides with the classical Bloch's higher Chow group. Taking a pair  $(X \times \mathbb{A}^1, X \times \{0\})$ , the higher Chow group with modulus agrees with the additive higher Chow group, which has been studied by Bloch, Esnault, Rülling, Park et al. In this talk, we formulate a generalization of  $\mathbb{A}^1$ -homotopy invariance of Bloch's higher Chow group to an invariance property of higher Chow groups with modulus, called cube invariance, and provide some applications.

望月 哲史/Satoshi Mochizuki (中央大学/Chuo University)

A dévissage theorem

In the lecture, we will show a version of dévissage theorem which provides an affirmative answer to the problem proposed by R. B. Thomason twenty six years ago. Our proof of the theorem is straightforward whenever we formulate a 'derived dévissage conditions' and however, in my experience, it can not seem to be expressed in the existing category theory. To overcome this stubborn, we will intensify the category theory by turning on the noiton of AQ structures. As an application of our theorem, we will figure out how to make motives of nonreduced schemes and  $\mathbb{A}^1$ -homotopy invariance phenomena of motives coexist in the realm of motives.

中屋 智瑛/Tomoaki Nakaya (九州大学/Kyushu University)

On modular solutions of certain modular linear differential equation and supersingular polynomials

Let  $M_k$  denote the space of modular forms of weight  $k$  on  $SL_2(\mathbb{Z})$ . Kaneko and Zagier introduced a certain differential equation from an endomorphism of  $M_k$ . Modular solution of that equation can be represented by Hypergeometric series and gives supersingular polynomial. In this talk, we construct an another endomorphism of  $M_k$  and prove that solution of corresponding differential equation gives supersingular polynomial.

大川 幸男/Sachio Ohkawa (東京大学/University of Tokyo)

Riemann-Hilbert correspondence for unit  $F$ -crystals on embeddable algebraic varieties.

For a separated scheme  $X$  of finite type over a perfect field  $k$  of characteristic  $p > 0$  which admits an immersion into a proper smooth scheme over the truncated Witt ring  $W_n$ , we define the bounded derived category of locally finitely generated unit  $F$ -crystals with finite Tordimension on  $X$  over  $W_n$ , independently of the choice of the immersion. Then we prove the anti-equivalence of this category with the bounded derived category of constructible étale sheaves of  $\mathbb{Z}/p^n\mathbb{Z}$ -modules with finite Tor dimension. Our result is a generalization of the Riemann-Hilbert correspondence for unit  $F$ -crystals due to Emerton-Kisin to the case of (possibly singular) embeddable algebraic varieties in characteristic  $p > 0$ .

大井 雅雄/Masao Ohi (東京大学/University of Tokyo)

単純超尖点表現のエンドスコピー持ち上げについて

On the endoscopic lifting of simple supercuspidal representations

Recently, Arthur proved the local Langlands correspondence for quasi-split classical groups over  $p$ -adic fields. He characterized the correspondence via the endoscopic character relation, which is an equality between the characters of representations of a classical group and the twisted characters of their liftings to a general linear group. In this talk, I will determine the liftings of simple supercuspidal representations of an odd special orthogonal group  $\mathrm{SO}(2n+1)$  to  $\mathrm{GL}(2n)$  explicitly by computing their characters and using the endoscopic character relation.

奥村 喜晶/Yoshiaki Okumura (東京工業大学/Tokyo Institute of Technology)

Non-existence of certain abelian varieties with CM factors

Rasmussen and Tamagawa stated the finiteness conjecture on abelian varieties over number fields with constrained prime power torsion. In a recent paper, they also stated a uniform version of this conjecture. In this talk, we show that the uniform version conjecture reduces to the absolutely simple abelian varieties case. By applying this result, we prove the uniform version conjecture for abelian varieties which have a  $\mathbb{Q}$ -simple factor with complex multiplication.

小田部 秀介/Shusuke Otabe (東北大学/Tohoku University)

Semi-étale finite bundles and its fundamental group

Nori introduced the notion of essentially finite bundles, as a generalization of (Weil-)finite bundles, on a proper variety  $X$  over a field. The fundamental group of the category of essentially finite bundles is called Nori fundamental group. In characteristic 0 case, it gives a reconstruction of Grothendieck's étale fundamental group. In this talk, I will introduce the notion of semi-étale finite bundles and consider its fundamental group, which is a hybrid of Grothendieck's étale fundamental group and Nori's unipotent fundamental group and, in characteristic 0 case, gives the virtual-unipotent fundamental group in the sense of Borne-Vistoli. I will explain some basic facts about this fundamental group.

小澤 友美/Tomomi Ozawa (東北大学/Tohoku University)

Classical weight one Hilbert cusp forms in a Hida family

Let  $F$  be a totally real field and  $p$  an odd prime. We consider a primitive  $p$ -ordinary Hida family  $\mathcal{F}$  of parallel weight Hilbert cusp forms defined over  $F$ . It is known that a specialization of  $\mathcal{F}$  at any arithmetic points of weight at least two is a classical Hilbert cusp form. However, this is not always the case for weight one specializations. Balasubramanyam, Ghate and Vatsal proved that such a Hida family admits infinitely many classical weight one specializations if and only if it is of CM type. In this talk, for a non-CM primitive  $p$ -ordinary Hida family  $\mathcal{F}$ , we give an explicit estimate on the number of classical weight one specializations of  $\mathcal{F}$ .

佐野 薫/Kaoru Sano (京都大学/Kyoto University)

積多様体の自己射に対する力学系次数と算術次数について

Dynamical Degree and Arithmetic Degree of Endomorphisms on Projective Varieties

My research is in the field of the arithmetic of dynamical systems. Let  $X$  be a smooth projective variety,  $f$  a rational self-map on  $X$ , and  $H$  an ample divisor on  $X$  all are defined over a number field  $k$ . The dynamical degree and the arithmetic degree are studied in the arithmetic of dynamical systems. The (first) *dynamical degree* of  $f$  is defined by

$$\delta_f := \lim_{n \rightarrow \infty} \deg((f^n)^* H) \cdot H^{d-1})^{1/n}.$$

On the other hand, the *arithmetic degree* at  $P$  is defined by

$$\alpha_f(P) = \lim_{n \rightarrow \infty} h_H^+(f^n(P))^{1/n}$$

for a point  $P \in X(\bar{k})$  at which  $f^n$  is defined for all  $n > 0$ , where  $h_H$  is the Weil canonical height associated with  $H$  and  $h_H^+ := \max\{h_H, 1\}$ . (Note that convergence of this limit is a conjecture in general.) Shu Kawaguchi and Joseph H. Silverman formulated the following conjecture.

Conjecture (Kawaguchi-Silverman) For every  $\bar{k}$ -rational point  $P \in X(\bar{k})$  whose forward  $f$ -orbit  $\mathcal{O}_f(P)$  is well-defined and Zariski dense in  $X$ , we have

$$\alpha_f(P) = \delta_f.$$

We explain the proof of Conjecture in the case of any surjective endomorphisms on product varieties of some varieties for which Conjecture is proved.

澤田 晃一郎/Koichiro Sawada (京都大学/Kyoto University)

多重双曲的曲線の遠アーベル幾何の副  $p$  およびコホモロジー的側面

Pro- $p$  and cohomological aspects of anabelian geometry of hyperbolic polycurves

A hyperbolic polycurve is a successive extension of families of hyperbolic curves. Mochizuki and Hoshi proved the Grothendieck conjecture for hyperbolic polycurves of dimension  $\leq 4$ . In the first part of this talk, we discuss the pro- $p$  version of this result. In the second part of the talk, we study cohomological properties of the fundamental group of hyperbolic polycurves. Among others, we show that the dimension of a hyperbolic polycurve is reconstructed group-theoretically from its fundamental group.

時本 一樹/Kazuki Tokimoto (東京大学/University of Tokyo)

Lubin-Tate パーフェクトイド空間のアフィノイドと局所 Langlands 対応の特別な場合について

Affinoids in the Lubin-Tate perfectoid space and special cases of the local Langlands correspondence

Let  $K$  be a non-archimedean local field and  $n$  a positive integer. The non-abelian Lubin-Tate theory asserts that the cohomology of the Lubin-Tate tower realizes the local Langlands correspondence for  $\mathrm{GL}_n(K)$  and the local Jacquet-Langlands correspondence. Motivated by this fact, Boyarchenko-Weinstein and Imai-Tsushima constructed a family of affinoids in the Lubin-Tate perfectoid space (a certain limit space of the tower) and proved that the cohomology of the reduction of (a formal model of) each affinoid realizes the two correspondences for some representations.

In this talk, we will discuss a similar result for some other representations assuming that  $K$  is of equal-characteristic.

Yang Yu (京都大学/Kyoto University)

Local  $p$ -rank and semi-stable reduction of curves

In this talk, we investigate the local  $p$ -ranks of coverings of stable curves. Let  $f : Y \rightarrow X$  be a morphism of stable curves over a complete discrete valuation ring with algebraically closed residue field of characteristic  $p > 0$ ,  $x$  a singular point of the special fiber  $X_s$  of  $X$ . Suppose that the generic fiber  $X_\eta$  of  $X$  is smooth, and the morphism of generic fibers  $f_\eta$  is a Galois étale covering with Galois group  $G$ . Write  $Y'$  for the normalization of  $X$  in the function field of  $Y$ ,  $\psi : Y' \rightarrow X$  for the resulting normalization morphism and  $y' \in \psi^{-1}(x)$  for a point of the inverse image of  $x$ . Suppose that the inertia group  $I_{y'} \subseteq G$  of  $y'$  is an abelian  $p$ -group. Then we give an explicit formula for the  $p$ -rank of a connected component of  $f^{-1}(x)$ . Furthermore, we prove that the  $p$ -rank is bounded by  $\#I_{y'} - 1$ , where  $\#I_{y'}$  denotes the order of  $I_{y'}$ .