AUTOMORPHISMS OF SUPERSINGULAR K3 SURFACES AND SALEM POLYNOMIALS: COMPUTATIONAL DATA

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We present the following data. These data are used for the proof of Theorems 1.2 and 1.3 in the paper "Automorphisms of supersingular K3 surfaces and Salem polynomials".

We use the notation defined in this paper. Let $X$ be a supersingular K3 surface in characteristic $p = p$ with Artin invariant $\sigma = \text{sigma}$.

- $\text{GramSX}[p, \text{sigma}]$ is a Gram matrix of the lattice $\Lambda_{p, \sigma}^-$, which is isomorphic to $S_X$.
- $h_0[p, \text{sigma}]$ is a vector $h_0$ of $\Lambda_{p, \sigma}^-$ with $\langle h_0, h_0 \rangle_A > 0$.
- $\text{Rh0}[p, \text{sigma}]$ is the set $\mathcal{R}(h_0)$.
- $\text{amplelist}[p, \text{sigma}]$ is an ample list of vectors $a = [h_0, \rho_1, \ldots, \rho_K]$. We identify $D(a)$ with $N(X)$ by a suitable isometry $\Lambda_{p, \sigma}^- \cong S_X$.
- $\text{sizeMs}[p, \text{sigma}]$ is the length $l$ of the list $[M(h_1), \ldots, M(h_l)]$ of matrix representations of double plane involutions $\tau(h_i) \in \text{Aut}(X)$ whose product $\tau(h_1) \cdots \tau(h_l)$ is of irreducible Salem type.

For $i = 1, \ldots, l = \text{sizeMs}[p, \text{sigma}]$, we present the following data.

- $h[p, \text{sigma}, i]$ is the polarization $h_i$ of degree 2 written as a row vector with respect to the basis of $\Lambda_{p, \sigma}^- \cong S_X$ that gives rise to the Gram matrix $\text{GramSX}[p, \text{sigma}]$.
- $\text{Exc}[p, \text{sigma}, i]$ is the data of the smooth rational curves contracted by $\Phi_{h_i} : X \to \mathbb{P}^2$.
- $\text{M}[p, \text{sigma}, i]$ is the matrix representation $M(h_i)$ on $S_X$ of the double plane involution $\tau(h_i) \in \text{Aut}(X)$.

The data $\text{Exc}[p, \text{sigma}, i]$ is a list of lists of the form $[\text{ADEtype}, \text{Cs}]$, each of which gives the information of a singular point $P$ of $B_{h_i}$. $\text{ADEtype}$ is the $ADE$-type of the singularity of $P$, and $\text{Cs}$ is the list of the classes of smooth rational curves contracted to $P$ by $\Phi_{h_i} : X \to \mathbb{P}^2$. These classes are sorted according to Figure 3.1 of the paper. If $B_{h_i}$ is nonsingular, then $\text{Exc}[p, \text{sigma}, i]$ is the empty list $\{\}$.

The following data gives the matrix representation $M$ of the automorphism $\tau(h_1) \cdots \tau(h_l)$ of irreducible Salem type.

- $\text{SalemM}[p, \text{sigma}]$ is the matrix

$$M := M(h_1) \cdots M(h_l),$$

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where \( l \) is \( \text{sizeMs}[p, \sigma] \).

- \( \text{charpolSalemM}[p, \sigma] \) is the characteristic polynomial \( \phi_M(t) \) of \( M \), which is a Salem polynomial of degree 22.
- \( \text{SalemNumb}[p, \sigma] \) is the real root of \( \phi_M(t) \) larger than 1 in the floating-point number expression.

The files are divided as follows. (IS stands for IrreducibleSalem.)

- \( \text{compdataIS0.txt} \): the data for \( p \) with \( 3 \leq p < 100 \) and \( \sigma \) arbitrary.
- \( \text{compdataIS1.txt} \): the data for \( p \) with \( 100 < p < 200 \) and \( \sigma \) arbitrary.
- \( \text{compdataIS2.txt} \): the data for \( p \) with \( 200 < p < 300 \) and \( \sigma \) arbitrary.
  - ... 
- \( \text{compdataIS79.txt} \): the data for \( p \) with \( 7900 < p \leq 7919 \) and \( \sigma \) arbitrary.
- \( \text{sigma10IS7.txt} \): the data for \( p \) with \( 7919 < p < 8000 \) and \( \sigma = 10 \).
- \( \text{sigma10IS8.txt} \): the data for \( p \) with \( 8000 < p < 9000 \) and \( \sigma = 10 \).
- \( \text{sigma10IS9.txt} \): the data for \( p \) with \( 9000 < p < 10000 \) and \( \sigma = 10 \).
  - ... 
- \( \text{sigma10IS17.txt} \): the data for \( p \) with \( 17000 < p \leq 17389 \) and \( \sigma = 10 \).

These files are zipped in \( \text{compdataIS.zip} \).