GROUP ACTIONS ON SYMMETRIC SPACES RELATED TO LEFT-INVARIANT GEOMETRIC STRUCTURES

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Abstract. We propose several problems on groups actions on symmetric spaces, the answers of which would give advantages for the study of left-invariant geometric structures.

We here list several problems on group actions on symmetric spaces. Some easy examples and applications will be mentioned in the talk.

Problem 1. Let $M = G/K$ be a Riemannian symmetric space of noncompact type, and consider an isometric action of H on M. Then, what the orbit space $H\backslash M$ can be?

For cohomogeneity one actions with H being connected, it is known that the orbit space $H\backslash M$ is homeomorphic to either $\mathbb{R}$ or $[0, +\infty)$. In our study on left-invariant Riemannian metrics on some Lie groups, this fact is quite suggestive.

Problem 2. Let $M = G/K$ be a Riemannian symmetric space of noncompact type, and consider an isometric action of H on M. Then, study the geometry of the orbits. Are there some "distinguished" orbits?

One knows the answers for cohomogeneity one actions with H connected and M irreducible. In fact, there are three types of such actions. This fits very nicely to "algebraic Ricci solitons" on three-dimensional solvable Lie algebras.

Problem 3. Let $M = L/Q$ be an R-space, where L is semisimple and Q is parabolic, and consider a subgroup H of L. Then, study the action of H on M. In particular, what happens if (L, H) is a symmetric pair?

In this talk, we mention that the action of $\mathrm{SO}(p, q)$ on $\mathbb{R}\mathrm{P}^{p+q-1}$ has exactly three orbits. This easy example derives a new result on left-invariant pseudo-Riemannian metrics on particular Lie groups.

Problem 4. Let $M = G/K$ be a pseudo-Riemannian symmetric space, and consider an isometric action of H on M. Then, study the action of H on M. In particular, what happens if H is parabolic?

In order to study left-invariant pseudo-Riemannian metrics on Lie groups, it would be an advantage to know the properties of such actions. Conversely, the study of the geometry of (solvable) Lie groups would give toy models.

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