Point Sets Constructed via a Linear Recurrence over F_{2^w}

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In a recent paper [1], we used a linear recurrence of order r over F_{2^w}, the finite field with 2^w elements, to obtain good pseudo-random number generators. In this presentation, we explore the usage of the same linear recurrences for quasi-Monte Carlo integration. With REGPOLY [2], we perform searches for good point sets with respect to equidistribution criteria. We also look at the uniformity on preselected small dimension projections. The point sets constructed are of infinite dimension and are dimension-stationnary, which means that the projections $\{i_1, i_2, \ldots, i_s\}$ and $\{i_1 + j, i_2 + j, \ldots, i_s + j\}$ in s dimensions, for all $j > 0$, are exactly the same. By looking at the quality of one projection, in fact, we assess an infinite number of projections. This helps in reducing the time needed to analyze the uniformity of a given point set and gives us guarantees on the quality of a large subset of all the projections possible. For the point sets considered, under certain restrictions on the parameters, the proportion of two-dimensional projections that are not perfectly equidistributed is exactly $(2^r - 1)/(2^{rw} - 1)$. We give sets of parameters for good point sets and compare their performance with other well-known constructions.

REFERENCES

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