

J.I. Cogolludo-Agustín The Cohomology Algebra of a Plane Curve

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## Recent Progress on Topology of Plane Curves: A Quick Trip Part II: The Cohomology Algebra of a Plane Curve

#### José Ignacio COGOLLUDO-AGUSTÍN

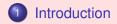
Departamento de Matemáticas Universidad de Zaragoza

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- Log-resolution Logarithmic Forms

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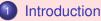
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Settings

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 $\mathcal{C} = \mathcal{C}_0 \cup \mathcal{C}_1 \cup ... \cup \mathcal{C}_r \subset \mathbb{P}^2$ 

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• Give a constructive description of  $H^*(X)$  by generators and relations, as well as describe the product.

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- Give a constructive description of H<sup>\*</sup>(X) by generators and relations, as well as describe the product.
- Weak Combinatorial Invariants of C.

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- Existence of an Orlik-Solomon-like algebra.
- Prove Formality of X.

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## The Line Arrangement Case

$$\mathcal{C} = \ell_0 \cup \ell_1 \cup \ldots \cup \ell_r \subset \mathbb{P}^2,$$

where  $\ell_i$  is a line.

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### The Line Arrangement Case

$$\mathcal{C} = \ell_0 \cup \ell_1 \cup ... \cup \ell_r \subset \mathbb{P}^2,$$

where  $\ell_i$  is a line. Consider  $X = \mathbb{C}^2 \setminus (\ell_1 \cup ... \cup \ell_r)$ .

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## The Line Arrangement Case

Theorem (Arnold, Brieskorn, Orlik-Solomon)

The ring  $H^*(X)$  is generated by  $H^1(X)$ , that is, by:

$$\sigma_i := \frac{d\ell_i}{\ell_i}$$

A complete set of relations is given by:

$$\sigma_i \wedge \sigma_j + \sigma_j \wedge \sigma_k + \sigma_k \wedge \sigma_i = \mathbf{0},$$

whenever  $\ell_i \cap \ell_j \cap \ell_k \neq \emptyset$ .

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### The Line Arrangement Case

Note that whenever  $\ell_i \cap \ell_j \cap \ell_k \neq \emptyset \Rightarrow \ell_k = a\ell_i + b\ell_j$ 

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$$\ell_i\ell_j\ell_k\cdot\sigma_j\wedge\sigma_k=a\ell_i(d\ell_j\wedge d\ell_i)$$

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$$\ell_i \ell_j \ell_k \cdot \sigma_k \wedge \sigma_i = b \ell_j (d \ell_j \wedge d \ell_i)$$

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$$\ell_i \ell_j \ell_k \cdot \sigma_j \wedge \sigma_k = a \ell_i (d \ell_j \wedge d \ell_i)$$

$$\ell_i\ell_j\ell_k\cdot\sigma_k\wedge\sigma_i=b\ell_j(d\ell_j\wedge d\ell_i)$$

Therefore,

$$\ell_i\ell_j\ell_k\cdot(\sigma_j\wedge\sigma_k+\sigma_k\wedge\sigma_i)=\ell_k(\mathbf{d}\ell_j\wedge\mathbf{d}\ell_i)=-\ell_i\ell_j\ell_k\cdot\sigma_i\wedge\sigma_j$$

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### The General Case

However,

$$\mathcal{C} = \ell_0 \cup q,$$

where  $\ell_0 = \{z = 0\}$  and  $q := \{z^2 = xy\}$ .

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## The General Case

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Therefore

 $\wedge^2 H^1(X) \neq H^2(X).$ 

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## The General Case

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$$H^2(X) = H_1(\mathcal{C}) = \mathbb{C}$$

Therefore

 $\wedge^2 H^1(X) \neq H^2(X).$ 

In fact,

$$H^2(X) = \langle \frac{\omega}{\ell_0 q} \rangle_{\mathbb{C}}, \quad \text{where } \omega := z dx \wedge dy + x dy \wedge dz + y dz \wedge dx.$$

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 $\pi: S \rightarrow \mathbb{P}^2$ 

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Definitions

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$$\begin{array}{rcccc} \pi : & \mathcal{S} & \to & \mathbb{P}^2 \\ & \cup & & \cup \\ & \bar{\mathcal{C}} & \to & \mathcal{C} \end{array} \tag{1}$$

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#### Definition

The sheaf  $\pi_* \mathcal{E}^*_{\mathcal{S}}(\log \overline{\mathcal{C}})$  is the sheaf of *log-resolution logarithmic* forms of  $\mathcal{C}$  w.r.t.  $\pi$ .

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$$\begin{array}{rcccc} \pi : & \mathcal{S} & \to & \mathbb{P}^2 \\ & \cup & & \cup \\ & \bar{\mathcal{C}} & \to & \mathcal{C} \end{array} \tag{1}$$

#### Definition

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#### Remark

• The sheaf  $\pi_* \mathcal{E}^*_{S}(\log \overline{C})$  is independent of the resolution.

Definitions

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$$\begin{aligned} \pi : & \boldsymbol{S} & \to & \mathbb{P}^2 \\ & \cup & & \cup \\ & \bar{\mathcal{C}} & \to & \mathcal{C} \end{aligned}$$
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## Definition

The sheaf  $\pi_* \mathcal{E}^*_{\mathcal{S}}(\log \overline{\mathcal{C}})$  is the sheaf of *log-resolution logarithmic* forms of  $\mathcal{C}$  w.r.t.  $\pi$ .

### Remark

- The sheaf π<sub>\*</sub> ε<sup>\*</sup><sub>S</sub>(log C
   ) is independent of the resolution.
- Denote it by  $\mathcal{E}^*_{\mathbb{P}^2}(\log \mathcal{C})$ .

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### Remark

- The sheaf π<sub>\*</sub> ε<sup>\*</sup><sub>S</sub>(log C
   ) is independent of the resolution.
- Denote it by *E*<sup>\*</sup><sub>ℙ<sup>2</sup></sub>(log *C*).

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# $H^{i}(S; W_{i}\mathcal{E}^{*}_{S}(\log \bar{\mathcal{C}}))$

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$$\begin{array}{c} H^{i}(\mathbb{P}^{2}; \, W_{i}\mathcal{E}_{\mathbb{P}^{2}}^{*}(\log \mathcal{C})) \\ \| \\ H^{i}(S; \, W_{i}\mathcal{E}_{S}^{*}(\log \bar{\mathcal{C}})) \end{array}$$

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$$\begin{array}{c} H^{i}(\mathbb{P}^{2}; W_{i}\mathcal{E}_{\mathbb{P}^{2}}^{*}(\log \mathcal{C})) \\ \parallel \\ H^{i}(S; W_{i}\mathcal{E}_{S}^{*}(\log \bar{\mathcal{C}})) \\ \parallel \\ H^{i}(X) \end{array}$$

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$$egin{aligned} & \mathcal{H}^i(\mathbb{P}^2; \, \mathcal{W}_i \mathcal{E}^*_{\mathbb{P}^2}(\log \mathcal{C})) & & \parallel \ & \mathcal{H}^i(S; \, \mathcal{W}_i \mathcal{E}^*_S(\log ar{\mathcal{C}})) & o & \mathcal{H}^i(S; \, \mathcal{W}_i / \mathcal{W}_{i-1}) \ & \parallel \ & \mathcal{H}^i(X) \end{aligned}$$

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$$egin{aligned} &\mathcal{H}^i(\mathbb{P}^2; \, \mathcal{W}_i \mathcal{E}^*_{\mathbb{P}^2}(\log \mathcal{C})) & & \parallel \ & \mathcal{H}^i(S; \, \mathcal{W}_i \mathcal{E}^*_{\mathcal{S}}(\log ar{\mathcal{C}})) & o & \mathcal{H}^i(S; \, \mathcal{W}_i / \mathcal{W}_{i-1}) &\simeq & \mathcal{H}^0(ar{\mathcal{C}}^{[i]}) & \ & \parallel \ & \mathcal{H}^i(X) \end{aligned}$$

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$$\begin{array}{ccc} H^{i}(\mathbb{P}^{2}; W_{i}\mathcal{E}_{\mathbb{P}^{2}}^{*}(\log \mathcal{C})) \\ & \parallel \\ H^{i}(S; W_{i}\mathcal{E}_{S}^{*}(\log \bar{\mathcal{C}})) & \rightarrow & H^{i}(S; W_{i}/W_{i-1}) & \simeq & H^{0}(\bar{\mathcal{C}}^{[i]}) \\ & \parallel \\ & H^{i}(X) \end{array}$$

Such a residue map will be denoted by  $\operatorname{Res}^{[i]}$ .

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$$\begin{array}{ccc} H^{i}(\mathbb{P}^{2}; W_{i}\mathcal{E}_{\mathbb{P}^{2}}^{*}(\log \mathcal{C})) \\ \| \\ H^{i}(S; W_{i}\mathcal{E}_{S}^{*}(\log \bar{\mathcal{C}})) & \rightarrow & H^{i}(S; W_{i}/W_{i-1}) & \simeq & H^{0}(\bar{\mathcal{C}}^{[i]}) \\ \| \\ H^{i}(X) \end{array}$$

Such a residue map will be denoted by Res<sup>[*i*]</sup>. In more generality:

$$H^{i}(\mathbb{P}^{2}; W_{k}\mathcal{E}^{*}_{\mathbb{P}^{2}}(\log \mathcal{C})) \xrightarrow{\mathsf{Res}^{[i,k]}} H^{i-k}(\overline{\mathcal{C}}^{[k]}).$$

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## Theorem (-,D.Matei)

Under the above conditions:

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## Theorem (-,D.Matei)

Under the above conditions:

•  $\operatorname{Res}^{[1,1]}$  is injective.

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## Theorem (-,D.Matei)

Under the above conditions:

Res<sup>[1,1]</sup> is injective.

If  $\psi \in \mathcal{E}^2(\mathbb{P}^2)(\log \mathcal{C})$  is such that  $\operatorname{Res}^{[2,2]} \psi = 0$  and  $\operatorname{Res}^{[2,1]} \psi = 0$ , then  $\psi = 0$ .

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# Example

Consider 
$$f = y^2 - x^3$$
,  $C = \{f = 0\}$ , and the 2-form  $\frac{dx \wedge dy}{f}$ .

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# Example

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$$\frac{dx \wedge dy}{f} \stackrel{x=u_1}{\longleftarrow} \frac{du_1 \wedge dv_1}{u_1(v_1^2 - u_1)}$$

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Consider 
$$f = y^2 - x^3$$
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$$\frac{dx \wedge dy}{f} \stackrel{y=u_1v_1}{\leftarrow} \frac{du_1 \wedge dv_1}{u_1(v_1^2 - u_1)}$$

$$\stackrel{u_1=u_2v_2}{\leftarrow} \frac{du_2 \wedge dv_2}{u_2v_2(v_2 - u_2)}$$

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$$\frac{dx \wedge dy}{f} \stackrel{\substack{x=u_1\\y=u_1v_1}}{\longleftarrow} \frac{du_1 \wedge dv_1}{u_1(v_1^2 - u_1)}$$

$$\stackrel{u_1=u_2v_2}{\longleftarrow} \frac{du_2 \wedge dv_2}{u_2v_2(v_2 - u_2)} \stackrel{\substack{u_2=u_3v_3\\y=u_2v_3}}{\longleftarrow} \frac{du_3 \wedge dv_3}{u_3v_3^2(1 - u_3)}$$

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which is not logarithmic.

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which is *not* logarithmic. However, if  $\psi = \varphi \frac{dx \wedge dy}{f}$ , then

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## Example

Consider 
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$$\frac{dx \wedge dy}{f} \stackrel{y=u_1v_1}{\leftarrow} \frac{du_1 \wedge dv_1}{u_1(v_1^2 - u_1)}$$

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which is *not* logarithmic. However, if  $\psi = \varphi \frac{dx \wedge dy}{f}$ , then •  $\varphi \in (x, y) \Rightarrow \psi \in \mathcal{E}_0^2(\log \mathcal{C})$ .

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## Example

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$$f = y^2 - x^3$$
,  $C = \{f = 0\}$ , and the 2-form  $\frac{dx \wedge dy}{f}$ .  

$$\frac{dx \wedge dy}{f} \stackrel{\substack{x=u_1\\y=u_1v_1}}{\longleftarrow} \frac{du_1 \wedge dv_1}{u_1(v_1^2 - u_1)}$$

$$\stackrel{u_1=u_2v_2}{\longleftarrow} \frac{du_2 \wedge dv_2}{u_2v_2(v_2 - u_2)} \stackrel{u_2=u_3v_3}{\longleftarrow} \frac{du_3 \wedge dv_3}{u_3v_3^2(1 - u_3)}$$

which is *not* logarithmic. However, if  $\psi = \varphi \frac{dx \wedge dy}{f}$ , then

- $\varphi \in (\mathbf{X}, \mathbf{Y}) \Rightarrow \psi \in \mathcal{E}_0^2(\log \mathcal{C}).$
- Moreover, if φ ∈ (y) ⇒ (Res<sup>[2,2]</sup> ψ)<sub>P</sub> = 0 at all P ∈ C
  <sup>[1]</sup> infinitely near 0.

Neak Combinatorics

## Theorem

The following is a presentation of  $H^*(X)$ :

• Generators in degree 1:  $\sigma_i$ , i = 1, ..., r,

Neak Combinatorics

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- Generators in degree 1:  $\sigma_i$ , i = 1, ..., r,
- Generators in degree 2:

$$\begin{array}{ll} \psi_{\mathcal{P}}^{\delta_1,\delta_2}, & \mathcal{P} \in \mathcal{C}_i \cap \mathcal{C}_j, \delta_1 \in \Delta_{\mathcal{P}}(\mathcal{C}_i), \delta_2 \in \Delta_{\mathcal{P}}(\mathcal{C}_j) \\ \psi_{\infty}^{i,k_i}, & i = 1,...,r, k_i = 1,..., d_i - 1 \\ \eta^{i,s_i}, \overline{\eta}^{i,s_i}, & i = 1,...,r, s_i = 1,..., g_i. \end{array}$$

Weak Combinatorics

#### Theorem

The following is a presentation of  $H^*(X)$ :

- Generators in degree 1:  $\sigma_i$ , i = 1, ..., r,
- Generators in degree 2:

$$\begin{array}{ll} \psi_{P}^{\delta_{1},\delta_{2}}, & P \in \mathcal{C}_{i} \cap \mathcal{C}_{j}, \delta_{1} \in \Delta_{P}(\mathcal{C}_{i}), \delta_{2} \in \Delta_{P}(\mathcal{C}_{j}) \\ \psi_{\infty}^{i,k_{i}}, & i = 1,...,r, k_{i} = 1,...,d_{i} - 1 \\ \eta^{i,s_{i}}, \overline{\eta}^{i,s_{i}}, & i = 1,...,r, s_{i} = 1,...,g_{i}. \end{array}$$

• Relations:

$$\begin{split} \psi_{P}^{\delta_{1},\delta_{2}} &= -\psi_{P}^{\delta_{2},\delta_{1}} \\ \psi_{P}^{\delta_{1},\delta_{2}} + \psi_{P}^{\delta_{2},\delta_{3}} + \psi_{P}^{\delta_{3},\delta_{1}} = \mathbf{0} \end{split}$$

for any  $P \in C_i \cap C_j \cap C_k$  and  $\delta_1 \in \Delta_P(C_i)$ ,  $\delta_2 \in \Delta_P(C_j)$ ,  $\delta_3 \in \Delta_P(C_k)$ .

Weak Combinatorics

## Theorem

The following is a presentation of  $H^*(X)$ :

•  $\sigma_i, \psi_P^{\delta_1,\delta_2}, \psi_\infty^{i,k_i}, \psi_i^{s_i}, \bar{\psi}_i^{s_i},$ 

$$\psi_P^{\delta_1,\delta_2} = -\psi_P^{\delta_2,\delta_1}$$

$$\psi_P^{\delta_1,\delta_2} + \psi_P^{\delta_2,\delta_3} + \psi_P^{\delta_3,\delta_1} = \mathbf{0}$$

• Product:

$$\sigma_i \wedge \sigma_j = \sum_{\boldsymbol{P} \in \mathcal{C}_i \cap \mathcal{C}_j} \mu_{\boldsymbol{P}}(\delta_1, \delta_2) \psi_{\boldsymbol{P}}^{\delta_1, \delta_2} + \boldsymbol{d}_i \sum_{k_j=1}^{d_j-1} \psi_{\infty}^{j, k_j} - \boldsymbol{d}_j \sum_{k_j=1}^{d_i-1} \psi_{\infty}^{i, k_j}.$$

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Weak Combinatorics

#### Remark

Note that from the given presentation one can deduce that  $H^*(X)$  only depends on the following invariants of C:

$$(\{1,...,r\}, \mathcal{S} = \operatorname{Sing} \mathcal{C}, \{\Delta_{P}\}_{P \in \mathcal{S}}, \{\phi_{P}\}_{P \in \mathcal{S}}, \{\mu_{P}\}_{P \in \mathcal{S}})$$

such an ordered set of invariants of C will be referred to as the *Weak Combinatorics of* C.

Weak Combinatorics

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such an ordered set of invariants of C will be referred to as the *Weak Combinatorics of* C.

Hence

#### Theorem

The cohomology algebra of X only depends on its weak combinatorics.

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# Consider $\omega \in H^1(X)$ .

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# Consider $\omega \in H^1(X)$ .

$$0 \to H^0(X) = \mathbb{C} \xrightarrow{\bullet \wedge \omega} H^1(X) \xrightarrow{\bullet \wedge \omega} H^2(X) \to 0 \qquad (H^*(X), \wedge \omega)$$

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## Definition

The *i*-th Resonance Variety of X is defined as

$$\mathcal{R}^i(X) := \{\omega \in H^1(X) \mid h^1(H^*(X), \wedge \omega) \geq i\}$$

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#### Remark

Note that for any graded algebra  $A^*$  one can analogously define the *i*-th Resonance Variety  $\mathcal{R}^i(A)$  of  $A^*$ .

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#### Theorem

There is an Orlik-Solomon-like graded algebra  $A^*$  whose resonance varieties are isomorphic to  $\mathcal{R}^i(X)$ .

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#### Theorem

There is an Orlik-Solomon-like graded algebra  $A^*$  whose resonance varieties are isomorphic to  $\mathcal{R}^i(X)$ .

$$\mathbf{A}^{1} := \sum_{i=1}^{r} \sigma_{i} \mathbb{C} \quad \mathbf{A}^{2} := \sum_{\mathbf{P} \in \mathcal{S}} \frac{\bigwedge^{2} A_{\mathbf{P}}}{I_{\mathbf{P}}},$$

where

$${\mathcal A}_{\mathcal P} := \sum_{\delta \in \Delta_{\mathcal P}} \psi^\delta_{\mathcal P} \mathbb C$$

$$I_{\mathcal{P}} := \langle \psi_{\mathcal{P}}^{\delta_1} \wedge \psi_{\mathcal{P}}^{\delta_2} + \psi_{\mathcal{P}}^{\delta_2} \wedge \psi_{\mathcal{P}}^{\delta_3} + \psi_{\mathcal{P}}^{\delta_3} \wedge \psi_{\mathcal{P}}^{\delta_1} \rangle_{\mathbb{C}}$$

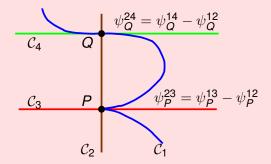
and

$$\sigma_i \wedge \sigma_j := \sum_{\mathcal{P} \in \mathcal{C}_i \cap \mathcal{C}_j} \mu_{\mathcal{P}}(\delta_1, \delta_2) \psi_{\mathcal{P}}^{\delta_1, \delta_2}$$

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Consider



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$$\sigma_{12} = 2\psi_P^{12} + \psi_Q^{12}$$
  

$$\sigma_{13} = 3\psi_P^{13}$$
  

$$\sigma_{14} = 3\psi_Q^{14}$$
  

$$\sigma_{23} = \psi_P^{13} - \psi_P^{12}$$
  

$$\sigma_{24} = \psi_Q^{24}$$
  

$$\sigma_{34} = \psi_R^{34}$$

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$$\begin{array}{l} \sigma_{12} = 2\psi_P^{12} + \psi_Q^{12} \\ \sigma_{13} = 3\psi_P^{13} \\ \sigma_{14} = 3\psi_Q^{14} \\ \sigma_{23} = \psi_P^{13} - \psi_P^{12} \\ \sigma_{24} = \psi_Q^{24} \\ \sigma_{34} = \psi_R^{34} \end{array} M := \begin{bmatrix} 2\beta & -2\alpha - \gamma & \beta & 0 \\ 3\gamma & \gamma & -3\alpha - \beta & 0 \\ \beta & -\alpha - \delta & 0 & \beta \\ 3\delta & \delta & 0 & -3\alpha - \beta \\ 0 & 0 & \delta & -\gamma \end{bmatrix}$$

• rank  $M = 2 \Leftrightarrow (\lambda, -3(\lambda + \mu), 2\mu, \mu)$ .

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$$\begin{array}{l} \sigma_{12} = 2\psi_P^{12} + \psi_Q^{12} \\ \sigma_{13} = 3\psi_P^{13} \\ \sigma_{14} = 3\psi_Q^{14} \\ \sigma_{23} = \psi_P^{13} - \psi_P^{12} \\ \sigma_{24} = \psi_Q^{24} \\ \sigma_{34} = \psi_R^{34} \end{array} \qquad M := \begin{bmatrix} 2\beta & -2\alpha - \gamma & \beta & 0 \\ 3\gamma & \gamma & -3\alpha - \beta & 0 \\ \beta & -\alpha - \delta & 0 & \beta \\ 3\delta & \delta & 0 & -3\alpha - \beta \\ 0 & 0 & \delta & -\gamma \end{bmatrix}$$

• rank  $M = 2 \Leftrightarrow (\lambda, -3(\lambda + \mu), 2\mu, \mu).$ 

• Notice that  $C_1$ ,  $3C_2$ ,  $2C_2 + C_4$  belong to a pencil of cubics.

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Max-Noether Fundamental Theorem Revisited

 A differential graded algebra (A, d<sub>A</sub>) is called *formal* if it has the same minimal model as its cohomology (H(A), 0).

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Max-Noether Fundamental Theorem Revisited

- A differential graded algebra (A, d<sub>A</sub>) is called *formal* if it has the same minimal model as its cohomology (H(A), 0).
- Since the minimal model of a d.g.a. is invariant under quasi-isomorphism, then it is more convenient to state that (A, d<sub>A</sub>) is formal if and only if there is a finite sequence of quasi-isomorphisms between (A, d<sub>A</sub>) and (H(A), 0).

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Max-Noether Fundamental Theorem Revisited

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#### Definition

A differential space X is called *formal* if its algebra of differential forms  $(\mathcal{E}(X), d)$  is formal.

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Aax-Noether Fundamental Theorem Revisited

#### Theorem (-, D. Matei, D. Macinic)

The complement of a plane curve X is a formal space.

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fax-Noether Fundamental Theorem Revisited

#### Theorem (-, D. Matei, D. Macinic)

The complement of a plane curve X is a formal space.

• 
$$(\mathcal{E}(X), d) \stackrel{q,i}{\simeq} (\mathcal{E}(\mathbb{P}^2)(\log \mathcal{C}), d),$$

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fax-Noether Fundamental Theorem Revisited

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$$(\mathcal{E}(X), d) \stackrel{q.i}{\simeq} (\mathcal{E}(\mathbb{P}^2)(\log \mathcal{C}), d),$$
  
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•  $(H(\mathcal{E}(X)), 0) \stackrel{q,i}{\simeq} (H(X), 0),$   
•  $H^*(X) \stackrel{e}{\to} \mathcal{E}^*(\mathbb{P}^2)(\log \mathcal{C})$ 

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Aax-Noether Fundamental Theorem Revisited

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$$\begin{array}{rcccc} H^*(X) & \stackrel{e}{\rightarrow} & \mathcal{E}^*(\mathbb{P}^2)(\log \mathcal{C}) \\ [\sigma_i] & \mapsto & \sigma_i \\ [\psi_P^{\delta_1,\delta_2}] & \mapsto & \psi_P^{\delta_1,\delta_2} \\ [\psi_\infty^{i,k_i}] & \mapsto & \psi_\infty^{i,k_i} \\ [\eta^{i,s_i}] & \mapsto & \eta^{i,s_i} \end{array}$$

fax-Noether Fundamental Theorem Revisited

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$$\begin{array}{cccc} H^*(X) & \stackrel{e}{\to} & \mathcal{E}^*(\mathbb{P}^2)(\log \mathcal{C}) \\ [\sigma_i] & \mapsto & \sigma_i \\ [\psi_{\mathcal{P}}^{\delta_1,\delta_2}] & \mapsto & \psi_{\mathcal{P}}^{\delta_1,\delta_2} \\ [\psi_{\infty}^{i,k_i}] & \mapsto & \psi_{\infty}^{i,k_i} \\ [\eta^{i,s_i}] & \mapsto & \eta^{i,s_i} \end{array}$$

Can we choose forms so that e is well-defined?

Max-Noether Fundamental Theorem Revisited

$$\psi_P^{\delta_1,\delta_2} + \psi_P^{\delta_2,\delta_3} + \psi_P^{\delta_3,\delta_1} = \mathbf{0}$$

Choose  $\delta_P$  at each  $P \in S$ , then

$$\psi_{P}^{\delta_{1},\delta_{2}} = \psi_{P}^{\delta_{P},\delta_{2}} - \psi_{P}^{\delta_{P},\delta_{1}}$$

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Max-Noether Fundamental Theorem Revisited

$$\sigma_i \wedge \sigma_j = \sum_{P \in \mathcal{C}_i \cap \mathcal{C}_j} \mu_P(\delta_i, \delta_j) \psi_P^{\delta_i, \delta_j} +$$

$$+d_i\sum_{k_j=1}^{d_j-1}\psi_{\infty}^{j,k_j}-d_j\sum_{k_i=1}^{d_i-1}\psi_{\infty}^{i,k_i}.$$

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Max-Noether Fundamental Theorem Revisited

$$\sigma_i \wedge \sigma_j =$$

$$= \sum_{P \in \mathcal{C}_i \cap \mathcal{C}_j} \mu_P(\delta_j, \mathcal{C}_i) \psi_P^{\delta_P, \delta_j} - \sum_{P \in \mathcal{C}_i \cap \mathcal{C}_j} \mu_P(\delta_i, \mathcal{C}_j) \psi_P^{\delta_P, \delta_i} +$$

$$+d_i\sum_{k_j=1}^{d_j-1}\psi_{\infty}^{j,k_j}-d_j\sum_{k_i=1}^{d_i-1}\psi_{\infty}^{i,k_i}.$$

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Max-Noether Fundamental Theorem Revisited

## Let $C_i, C_j, C_k$ be such that:

• 
$$d_i = d_j = d_k$$

• 
$$\mu_P(\delta_i, \mathcal{C}_j) = \mu_P(\delta_i, \mathcal{C}_k),$$

• 
$$\mu_P(\delta_j, C_i) = \mu_P(\delta_j, C_k),$$

• 
$$\mu_P(\delta_k, C_i) = \mu_P(\delta_k, C_j),$$

#### then

$$\sigma_i \wedge \sigma_j + \sigma_j \wedge \sigma_k + \sigma_k \wedge \sigma_i = 0 \tag{3}$$

Note that if  $C_k = \alpha C_i + \beta C_j$ , then (3) is trivial.

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Max-Noether Fundamental Theorem Revisited

Theorem (Max-Noether Fundamental Theorem (M.Noether,...,Fulton))

Let F, G, and H be three plane curves with no common components. If  $H_P \in (F_P, G_P)$  at any  $P \in V(F) \cap V(G)$ , then there exist two forms  $A, B \in \mathbb{C}[x, y, z]$  such that

H = AF + BG

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Max-Noether Fundamental Theorem Revisited

Theorem (Max-Noether Fundamental Theorem (M.Noether,...,Fulton))

Let F, G, and H be three plane curves with no common components. If  $H_P \in (F_P, G_P)$  at any  $P \in V(F) \cap V(G)$ , then there exist two forms  $A, B \in \mathbb{C}[x, y, z]$  such that

$$H = AF + BG$$

#### Remark

The conditions  $H_P \in (F_P, G_P)$  at any  $P \in V(F) \cap V(G)$  are commonly known as the *Noether Conditions*.

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Max-Noether Fundamental Theorem Revisited

## Definition

Three curves F, G, and H satisfying ( $\bigcirc$ ) are said to belong to a *combinatorial pencil*.

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Max-Noether Fundamental Theorem Revisited

#### Definition

Three curves F, G, and H satisfying ( $\bigcirc$ ) are said to belong to a *combinatorial pencil*.

### Theorem (-,M.A.Marco)

If F, G, and H belong to a primitive combinatorial pencil, then they belong to an algebraic pencil ( $H = \alpha F + \beta G$ ).

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Max-Noether Fundamental Theorem Revisited

## Remark

• The Noether Conditions can be replaced by the Combinatorial Conditions.

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Max-Noether Fundamental Theorem Revisited

## Remark

- The Noether Conditions can be replaced by the Combinatorial Conditions.
- Primitive translates into a minimality condition.

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Max-Noether Fundamental Theorem Revisited

### Remark

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## Proposition

Any combinatorial pencil admits a primitive refinement.

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Max-Noether Fundamental Theorem Revisited

### Remark

- The Noether Conditions can be replaced by the Combinatorial Conditions.
- Primitive translates into a minimality condition.

## Proposition

Any combinatorial pencil admits a primitive refinement.

This proves the formality of X.

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## **Open Problems**

• Are there also *nice* combinatorial descriptions of *H*<sup>\*</sup>(*X*) in higher dimensions?

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## **Open Problems**

- Are there also *nice* combinatorial descriptions of *H*<sup>\*</sup>(*X*) in higher dimensions?
- Are the complements of hypersurfaces in the projective space formal?

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# **Open Problems**

- Are there also *nice* combinatorial descriptions of *H*<sup>\*</sup>(*X*) in higher dimensions?
- Are the complements of hypersurfaces in the projective space formal?
- What about toric varieties, or weighted projective spaces?

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# **Open Problems**

- Are there also *nice* combinatorial descriptions of *H*<sup>\*</sup>(*X*) in higher dimensions?
- Are the complements of hypersurfaces in the projective space formal?
- What about toric varieties, or weighted projective spaces?
- Study the resonance varieties of Abstract Curve Combinatorics. This could lead to criteria for non-quasiprojective groups.