NON-KÄHLER COMPLEX STRUCTURES ON $\mathbb{R}^4$

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We consider the following natural problem. "Is there any non-Kähler complex structures on $\mathbb{R}^{2n}$?" If $n = 1$, there is no such complex structure. On the other hand, if $n \geq 3$, there exist uncountably many non-Kähler complex structures on $\mathbb{R}^{2n}$. It was proven by Calabi and Eckmann as the consequence of a simple application of the Calabi-Eckmann manifolds. In this talk, I will construct uncountably many non-Kähler complex structures on $\mathbb{R}^4$, and give the affirmative answer to the above problem for the case where $n = 2$.

The outline of the construction is as follows. First, we prepare two complex manifolds, a tubular neighborhood of a singular elliptic fiber of type I_1 and the product of a holomorphic disk and a holomorphic annulus, and glue them by a biholomorphism of certain gluing domains. Then, we show that the resultant complex manifold is diffeomorphic to $\mathbb{R}^4$ by using a genus-1 achiral Lefschetz fibration $S^4 \to S^2$ found by Yukio Matsumoto and Kenji Fukaya. It is non-Kähler, since it contains holomorphic tori which are homotopically trivial, by construction.

This is a joint work with Antonio J. Di Scala and Daniele Zuddas.