Title: Fibrations and Differential Equations on an Algebraic Surface

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Abstarct: In the 19th century, Darboux, Painlev\'e and Poincar\'e studied differential equations of the first order by using complex algebraic geometry. More precisely, the theory of integrable curves defined by complex differential equations is similar to the theory of families of algebraic curves, i.e., fibrations on an algebraic surface.   Poincar\'e proposed the following research program.

1) Study the (topological) properties of families of algebraic curves on a complex algebraic surface, and check if they are the properties of differential equations.
2) Find numerical invariants of complex differential equations.
3) Classify complex differential equations according to their invariants.
4) Characterize those complex differential equations which are algebraically integrable.
5) Apply to some problems on real differential equations.

In this lecture, I will talk about the recent progress in Poincar\'e Program.