Limits of periodic minimal surfaces

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Abstract

In this talk, we consider various families of periodic minimal surfaces in $\mathbb{R}^3$.

A minimal surface in $\mathbb{R}^3$ is said to be *periodic* if it is connected and invariant under a group Γ of isometries of $\mathbb{R}^3$ that acts properly discontinuously and freely. Γ can be chosen to be a rank three lattice Λ in $\mathbb{R}^3$ (the triply periodic case), a rank two lattice $\Lambda \subset \mathbb{R}^2 \times \{0\}$ generated by two linearly independent translations (the doubly periodic case), or a cyclic group Λ generated by a screw motion symmetry (the singly periodic case). The geometry of a periodic minimal surface can usually be described in terms of the geometry of its quotient surface in the flat three manifold $\mathbb{R}^3/\Lambda$. Hence a triply periodic minimal surface (TPMS) is a minimal surface in a flat torus $\mathbb{T}^3$, a doubly periodic minimal surface (DPMS) is a minimal surface in $\mathbb{T}^2 \times \mathbb{R}$, and a singly periodic minimal surface is a minimal surface in $S^1 \times \mathbb{R}^2$.

A (non-flat) properly immersed TPMS in $\mathbb{R}^3$ can be considered as a compact minimal surface of genus $g \geq 3$ in $\mathbb{T}^3$. We will focus on the genus-three case. It is known that a compact oriented minimal surface of genus three in a flat three-torus is hyperelliptic, that is, it can be represented as a two-sheeted branched covering of the sphere.

In this talk we study limits of a family of compact oriented embedded TPMS of genus three and show several results obtained in joint work with Norio Ejiri and Toshihiro Shoda, [1] and [2]. We exhibit various graphics of examples as well.

References

[1] N. Ejiri, S. Fujimori, and T. Shoda, *A remark on limits of triply periodic minimal surfaces of genus 3*, Topology Appl. **196** (2015), 880–903.

[2] N. Ejiri, S. Fujimori, and T. Shoda, *On limits of triply periodic minimal surfaces*, Ann. Mat. Pura Appl. **197** (2018), 1739–1748.