# Universal Convex Multipliers

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*Abstract*

It is well-known that the Hadamard convolution of two convex univalent functions in the unit disk of the complex plane is also convex univalent (Pólya-Schoenberg conjecture [1958], Ruscheweyh & Sheil-Small [1973]). We study the same situation for other discs/half-planes, and it turns out that the results are substantially different. In the 'universal' case, where convexity preservation with convex univalent functions in arbitrary disks and half-planes (containing the origin) is required we are led to a specific class of Pick functions, for which an almost explicit representation is derived. It is, however, a demanding problem to identify more explicit solutions to this problem. In particular, to decide which hypergeometric functions share this property seems to be a non-trivial task. Also the poly-logarithms are candidates for membership, awaiting verification.