# Hiroshima Mathematical Analysis Seminar No.148

Borel summability for formal solution of initial value problem
of some linear partial differential equations with variable coefficients

We study the following initial value problem:

$$
\begin{cases}
\frac{\partial}{\partial t}u(t,x) & = & (\frac{\partial}{\partial x})^2 u(t,x) + At(t\frac{\partial}{\partial t})^3 u(t,x) \\
u(0,x) & = & \phi(x)
\end{cases}
\tag{1}
$$

where $(t,x) \in \mathbb{C} \times \mathbb{C}$ and $A \in \mathbb{C}$.

We will consider a formal power series solution $\tilde{u}(t,x) = \sum_{i=0}^{\infty} u_i(x)t^i$ of the equation (1).

The formal solution $\tilde{u}(x,t)$ diverges for general initial value. In the case of $A = 0$, the equation (1) is the heat equation, and we have a result of Luts-Miyake-Schäfke [2] for Borel summability of the formal solution $\tilde{u}(t,x)$. For a general equation, we have a result of Balser [1] by using an idea of a normalized formal solution.

Here we consider Borel summability of the formal solution $\tilde{u}(t,x)$ in the case of $A \neq 0$. We get a result of Borel summability under the condition that the initial value $\phi(x)$ is an entire function of exponential order at most 2, that is, the following inequality holds for some positive constants $C$ and $c$,

$$|\phi(x)| \leq C \exp c|x|^2, \; x \in \mathbb{C}.$$

In the case without a term $(\frac{\partial}{\partial x})^2 u(t,x)$, we have a result of Ōuchi [3] and [4] for some inhomogeneous linear and nonlinear partial differential equations. Ōuchi showed (Multi) summability of the formal solution $\tilde{u}(t,x)$ by using the Borel transform and the Laplace transform.

Our proof method adopts some idea in [1] and [3].

[1]  W. Balser, summability of formal power-series solutions of partial differential equations with constant coefficients, *Journal of Math. Sci.*, Vol. 124, No. 4, (2004), 5085–5097.

[2]  D. A. Luts, M. Miyake and R. Schäfke, On the Borel sommability of divergent solutions of the heat equation, *Nagoya Math. J.* Vol., 154, (1999), 1–29.

[3]  S. Ōuchi, Multisummability of Formal Solutions of Some Linear Partial Differential Equations, *J. Diff. Eq.* **185** (2002), 513–549.

[4]  S. Ōuchi, Multisummability of formal power series solutions of nonlinear partial differential equations in complex domains, *Asymptotic Analysis* **47** (2006), 187–225.

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