Singular limit of a Fisher equation with degenerate diffusion

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The purpose of this study is to gain a better understanding of some reaction-diffusion models for bacterial colonies; such models often have the form of a noncooperative reaction-diffusion system with nonlinear diffusion and the main unknown functions are the density of a bacteria and the concentration of some nutrient.

In this talk we consider a simpler model involving a Fisher equation with degenerate diffusion, namely

$$u_t = \varepsilon \Delta u^m + \frac{1}{\varepsilon} u(1-u) \qquad m > 1,$$

together with a homogeneous Neumann boundary condition and a suitable initial condition. We prove that, as ε tends to zero, u^ε converges to a limiting function u which is almost everywhere equal to zero or to one and that the interface between the regions where $u = 0$ and $u = 1$ moves according to the law

$$V_n = c_*,$$

where c_* is the minimum velocity of the travelling waves of a related equation.