A formulation of the Poincaré Conjecture using knot groups

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An *Artin presentation* is a group presentation

$$(x_1, \dots, x_n : r_1, \dots, r_n)$$

such that

$$\prod_{i=1}^{n} r_i x_i r_i^{-1} = \prod_{i=1}^{n} x_i$$

in $F(x_1, \dots, x_n)$.

Definition. If $(x_1, \dots, x_n : r_1, \dots, r_n)$ is an Artin presentation of the trivial group, then $\|(x_1, \dots, x_n : r_1, \dots, r_{n-1})\|$ is a *RAT-group*.

Definition. A *knot group* is a group that has a presentation

$$(y_1, \dots y_n : s_1 y_1 s_1^{-1} y_2^{-1}, \dots, s_{n-1} y_{n-1} s_{n-1}^{-1} y_n^{-1}, s_n y_n s_n^{-1} y_1^{-1})$$

such that

$$\prod_{i=1}^{n} s_i y_i s_i^{-1} = y_2 y_3 \cdots y_n y_1$$

in $F(y_1, \dots, y_n)$.

Conjecture. RAT-groups are knot groups.

Theorem. This conjecture is equivalent to Poincaré's Conjecture.