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Longitudinal slope and Dehn fillings

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Abstract. Let M be an irreducible 3-manifold with an incompressible torus boundary T, and γ a slope on T, which bounds an incompressible surface, with genus g say. We assume that there exists a slope r that produces an essential 2-sphere by Dehn filling.

Let q be the minimal geometric intersection number between the essential 2-sphere and the core of the Dehn filling. Then, we show that $q = 2$ or the minimal geometric intersection number between γ and r is bounded by $2g - 1$.

In the special case that M is the exterior of a non-cable knot K in S^3, we show that $q \geq 6$ and $|r| \leq 2g - 1$, where g is the genus of the knot K. We get also similar and simpler results for the projective slopes. These imply immediately a known result that the cabling and $\mathbf{R}P^3$ conjectures are true for genus one knots.

1. Introduction

All 3-manifolds are assumed to be compact and orientable. Let M be a 3-manifold, with a torus T as boundary. *A slope* r on T is the isotopy class of an unoriented essential simple closed curve on T. The slopes are parametrized by $\mathbf{Q} \cup \{\infty\}$ (for more details, see [25]).

A *Dehn filling on* M is to glue a solid torus $V = S^1 \times D^2$ to M along T. We call it an r-*Dehn filling* when the attaching homeomorphism sends a meridian curve of ∂V to the slope r on T. We denote by $M(r)$ the resulting 3-manifold after the r-Dehn filling.

A 3-manifold is *reducible* if it contains an essential 2-sphere, that is, a 2-sphere which does not bound a 3-ball; otherwise it is an *irreducible* 3-manifold. A slope r in T is said to be a *reducing slope* if M is irreducible and $M(r)$ is reducible (that means that r produces an essential 2-sphere).

Similarly, a *projective slope* is a slope p that produces a projective plane by Dehn filling. This means that M does not contain a projective plane but $M(p)$ contains a projective plane.

Many papers focus on projective or reducing slopes:

i) There exist at most three reducing slopes (see [15, 19]) and three projective slopes (see [22, 28]);

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ii) M is not necessarily cabled, because there exists an infinite family of hyperbolic manifolds, which admit two reducing slopes (see [20]) and many of them are also projective slopes;

iii) When M is the exterior of a knot in S^3, reducing slopes (see [13]) and projective slopes (see the proof of Corollary 1.4 below) are integers; and there is at most one projective slope (see [22, 28]).

A slope γ on T is called a *longitudinal slope* if there exists an orientable surface F properly embedded in M, whose boundary is a loop having slope γ. In fact, for any such (M, T) there is at most one longitudinal slope (see [21, Lemma 8.1]).

Then the *genus* of γ is defined to be the minimal genus of such F.

Recall that *the distance* between two distinct slopes α and β is their minimal geometrical intersection number, denoted by $\Delta(\alpha, \beta)$.

The main result of this paper is the following:

THEOREM 1.1. *Let M be an irreducible* 3*-manifold with a torus T as boundary. Assume that M is not a solid torus. Let γ be a longitudinal slope, and g the genus of γ.*

i) *If there exists a reducing slope r, then $\Delta(r, \gamma) \leq 2g - 1$ or $q = 2$, where q is the minimal geometric intersection number between essential* 2*-spheres in $M(r)$ and the core of the r-Dehn filling.*

ii) *If there exists a projective slope p which is not a reducing slope, then $\Delta(p, \gamma) \leq 2g - 1$.*

COROLLARY 1.2. *If M is hyperbolic and θ is a reducing or projective slope, then $\Delta(\gamma, \theta) \leq 2g - 1$.*

PROOF. Assume that θ is a reducing slope. Recall that q is the minimal geometric intersection number between essential 2-spheres in $M(r)$ and the core of the r-Dehn filling.

If $q = 2$ then M contains an essential annulus, so M is Seifert fibered or toroidal. □

Note that the examples of infinite family of irreducible manifolds M, which admit two distinct reducing slopes (see [6, 20] for more details) are hyperbolic manifolds.

We consider now the case that M is the exterior $E(K)$ of a non-trivial knot in S^3. An *r-Dehn surgery on K* is an r-Dehn filling on $E(K)$. Concerning the existence of reducing or projective slopes, we have two famous following conjectures:

THE CABLING CONJECTURE (González-Acuña and Short [8]).

If a Dehn surgery on a non-trivial knot in S^3 produces a reducible manifold, then K is a cable knot.

THE $\mathbf{R}P^3$ CONJECTURE (Gordon [10]).
Any Dehn surgery on a non-trivial knot in S^3 cannot produce $\mathbf{R}P^3$.

We prove the followings:

PROPOSITION 1.3. *Let K be a non-trivial knot in S^3, and g be its genus.*
i) *Assume there exists a reducing slope r in $\partial E(K)$. Let q be the minimal geometric intersection number with essential 2-spheres in $E(K)(r)$ and the core of the r-Dehn surgery.*
If K is not a cable knot, then $q \geq 6$ and $|r| \leq 2g-1$.
ii) *Assume that there exists a projective slope p in $\partial E(K)$, which is not a reducing slope, then $|p| \leq 2g-1$.*

We can note that in case ii), all projective planes are pierced at least five times by the core of the Dehn surgery (see [5]). Consequently, the spheres, which are the 2-covering of them, are pierced at least ten times by the core of the Dehn surgery.

COROLLARY 1.4. *Genus one knots satisfy the cabling conjecture, and the $\mathbf{R}P^3$-conjecture.*

PROOF. Let K be a genus one knot, and let r be a reducing slope. If K is not a cable knot, then $|r| = 0$ or 1 by Proposition 1.3. But $E(K)(0)$ is irreducible by [7]. Also $E(K)(\pm 1)$ is an irreducible homology sphere by [14, Corollary 3.1]. This proves the cabling conjecture for genus one knots.

If p is a projective slope, which is not a reducing slope, then $E(K)(p) = \mathbf{R}P^3$. Since K is not a torus knot (by [23]), we obtain that p is an integer (by the cyclic surgery theorem, see [2]). Finally the first homology group of $E(K)(p)$ is $H_1(E(K)(p)) = \mathbf{Z}/p$. Therefore $p = 2 = 2/1$, which does not satisfy the inequality $2 \leq 2g-1$. $\square$

This corollary is also known by [1] for the cabling conjecture, and independently, by [3, 27] for the $\mathbf{R}P^3$ conjecture.

The core of the paper is divided into two parts. §2 concerns the general case of Dehn fillings, and the proof of the Theorem 1.1. §3 studies the special case of Dehn surgeries, and results towards the cabling conjecture, or the $\mathbf{R}P^3$ conjecture. In §4 we give comments and questions.

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2. Proof of Theorem 1.1

PROOF OF i)
Let P be an incompressible surface in M, properly embedded in M, such

that ∂P is one simple closed curve, representing the slope γ in T. Let g be the genus of P.

We suppose that T contains a reducing slope r. Let K_r be the core of the r-Dehn filling, and V_r the attached solid torus of the r-Dehn filling.

Let $\hat{Q}$ be a *minimal* essential 2-sphere in $M(r)$, that means that $\hat{Q}$ is pierced a minimal number of times by K_r, among all essential 2-spheres in $M(r)$.

Let q be the number of intersection between $\hat{Q}$ and the core of the r-Dehn surgery. Since M does not contain an essential 2-sphere, then q is a positive integer. Let $Q = \hat{Q} \cap M = \hat{Q} - \operatorname{int} V_r$.

If $q = 1$ then by the uniqueness of longitudinal slope, we have that $\gamma = r$ and so $\Delta(\gamma, r) = 0$. But the essential 2-sphere is non-separating, and so the knot is trivial by [7]. Therefore, we may assume that $q > 2$.

Now we consider the pair of intersection graphs (G_P, G_Q), which comes from the intersection of the surfaces P and Q in the usual way (see [9] for more details). We recall some basic definitions, useful for the following.

The (fat) vertices of G_Q are the disks $\hat{Q} - \operatorname{int} Q$. If we cap off the boundary component of P by a disk (which corresponds to a meridian disk of γ-Dehn filling) we obtain a closed surface $\hat{P}$. The disk $\hat{P} - \operatorname{int} P$ is the vertex of G_P.

The edges of G_P are the arc components of $P \cap Q$ in $\hat{P}$, and similarly the edges of G_Q are the arc components of $P \cap Q$ in $\hat{Q}$. We number the components of ∂T by $1, 2, \dots, q$ in the order in which they appear. This gives a numbering of the vertices of G_Q. Furthermore, it induces a labelling of the endpoints of the edges of G_P: the label at one endpoint of an edge corresponds to the number of the boundary component of Q that contains this endpoint.

Two vertices on any graph are said to be *parallel* if the ordering of the labels on each is the same (clockwise for example); otherwise the vertices are said to be *antiparallel*.

A Scharlemann cycle is a cycle σ which bounds a disk face, whose vertices are parallel, and such that the endpoints of the edges of σ have the same pair of labels. Consequently, any Scharlemann cycle has two successive labels, which are called *the labels of the Scharlemann cycle*.

A trivial loop is an edge that bounds a disk face.

CLAIM 2.1. *The graphs G_Q and G_P do not contain a trivial loop.*

PROOF. Since P is an incompressible and boundary incompressible surface, G_Q cannot contain a trivial loop.

Similarly, since $\hat{Q}$ is minimal and $q > 2$, it is also an incompressible and boundary incompressible surface. Therefore G_P cannot contain a trivial loop. □

Let x be a label of G_P. Note that G_P has only one vertex. Therefore, since $\hat{Q}$ is orientable, any edge in G_P cannot have the same label at both endpoints (by the parity rule). We denote by Γ_x the subgraph of G_P consisting of the unique vertex and the edges with one endpoint labelled by x.

Claim 2.2. *If $\Delta(\gamma, r) \geq 2g$ then Γ_x contains a disk face, for all labels x of G_P.*

Proof. The Euler characteristic calculation for Γ_x gives $\chi(\hat{P}) = 2 - 2g = V - E + F$, where V is the number of vertices, E is the number of edges of Γ_x, and $F = \sum_{f \text{ face of } \Gamma_x} \chi(f)$.

Since $V = 1$ and $E = \Delta(\gamma, r)$, we obtain that $F = 1 - 2g + \Delta(\gamma, r)$. Therefore, if $\Delta(\gamma, r) \geq 2g$ then $F \geq 1$, which means there exists a disk face in Γ_x. □

Assume for contradiction that $\Delta(\gamma, r) \geq 2g$, and that $q \geq 3$.

A *strict great cycle* is a great cycle which is not a Scharlemann cycle. From [18] a strict great cycle in G_P implies that $\hat{Q}$ is not minimal. More precisely, in [18] Hoffman proves that any strict great cycle contains *seemly pairs* ([18, Lemma 5.2]) and find a new essential 2-sphere, using the seemly pairs, which is pierced less than the first by the core of the surgery. We want to find seemly pairs, which represents a contradiction to the minimality of $\hat{Q}$.

Let $L = \{1, 2, \ldots, q\}$ be the set of labels of G_P. Then for each $x \in L$, Γ_x contains a disk face. Therefore G_P contains a Scharlemann cycle [16]. By [15, Theorem 2.4] all the Scharlemann cycles in G_P have the same labels. Without loss of generality, we may assume that $\{1, 2\}$ are the labels of the Scharlemann cycle.

We consider the graph Γ_3. Let D be a disk face of Γ_3. Since 3 is not the label of a Scharlemann cycle, D contains a seemly pair by [24], which gives the required contradiction.

Proof of ii)

Let $\hat{S}$ be a projective plane in $M(p)$ pierced a minimal number of times s by the core of the Dehn filling. If $s = 1$, then $S = \hat{S} \cap M$ is a Mobius band, and so M is a cabled manifold; therefore p is also a reducing slope or M is a solid torus. Thus, we may assume that $s \geq 2$. Now, we consider the 2-sphere $\hat{Q}$, which is the 2-covering of $\hat{S}$ in $M(p)$. Again, q is the intersection number between $\hat{Q}$ and the core of the p-Dehn filling. Since $\hat{Q}$ is the boundary of a thin regular neighbourhood of $\hat{S}$, we have that $q = 2s > 2$.

First, we consider the graphs that come from P and S. They cannot contain a trivial loop, by the minimality of S. Therefore, the graphs (G_P, G_Q), from P and Q, can also not contain a trivial loop.

We repeat exactly the same argument, as for the case i).

3. Proof of Proposition 1.3

Let P be an incompressible Seifert surface of K in S^3, and g be its genus. Then $\gamma = \partial\hat{P}$, where γ is the preferred longitudinal slope $\frac{0}{1}$ on $T_K = \partial E(K)$.

PROOF OF i)

Assume that there exists a reducing slope r on T_K. Let K_r be the core of the r-Dehn surgery, and V_r the attached solid torus of the r-Dehn surgery. Then $E(K)(r)$ is the union of $E(K)$ and V_r along their boundaries.

Let $\hat{Q}$ be a *minimal* essential 2-sphere in $E(K)(r)$, that means that $\hat{Q}$ is pierced a minimal number of times, q say, among all essential 2-spheres in $E(K)(r)$, by the core of the r-Dehn surgery. By [13] we know that r is an integer, so the minimal geometric intersection number between the slopes γ and r is $\Delta(\gamma, r) = |r|$.

Since $E(K)$ does not contain an essential 2-sphere, then q is a positive number. Recall that the essential 2-spheres in $E(K)(r)$ are separating. Indeed, by [7] $E(K)(0)$ is irreducible, so $r \neq 0$. Moreover, $H_1(E(K)(r)) = \mathbf{Z}/r\mathbf{Z}$, then any 2-sphere in $E(K)(r)$ is separating (otherwise $H_1(K(E)(r))$ should be infinite).

Consequently, $q \geq 2$ is an even integer.

Let $Q = \hat{Q} \cap E(K) = \hat{Q} - \text{int } V_r$.

By Theorem 1.1, we obtain that if $q \neq 2$ then $|r| \leq 2g - 1$.

If $q = 2$ then $E(K)$ is toroidal or Seifert fibered. Then K is respectively, a satellite knot or a torus knot. But these knots satisfy the cabling conjecture (see [26] and [23]). Therefore K is cabled.

So, we may assume that $q > 2$. Therefore $|r| \leq 2g - 1$.

CLAIM 3.1. $q \neq 4$.

PROOF. There exists a level 2-sphere $\hat{S}$ in S^3 corresponding to a thin position of K in S^3, so that (for more details, see [7]):

i) Boundary components of $S = \hat{S} \cap E(K)$ have slope ∞.

ii) S and Q intersect transversaly, and each component of ∂S meets each component of ∂Q in exactly one point (since the slope r is an integer slope).

iii) each arc component of $S \cap Q$ is essential in S and Q.

We consider the pair of intersection graphs (G_Q, G_S), which comes from the intersection of the surfaces Q and S in the usual way (see [9] for more details).

Since no arc component of $Q \cap S$ is boundary parallel in either S or Q, the graphs G_S and G_Q do not contain a trivial loop.

Since S^3 does not contain non-trivial torsions, G_Q does not represent all types (see [9, 14] for more details). Therefore, G_S contains a Scharlemann cycle σ ([14, Proposition 2.8.1]). Without loss of generality, we may assume that $\{1, 2\}$ are the labels of a Scharlemann cycle in G_S.

Assume now that $q = 4$. Let $\{3, 4\}$ be the two remaining labels of G_S. Let V_i be the vertex numbered by i in G_Q, for $i \in \{1, 2, 3, 4\}$. The edges of σ, with the vertices V_1 and V_2 partition $\hat{Q}$ into distinct disks, called *bigons*.

Subclaim 3.2. *The vertices V_3 and V_4 are in the same bigon.*

Proof. If V_3 and V_4 are not in the same bigon, then let B_i be the bigon which contains only the vertex V_i, for $i = 3, 4$. Since G_Q does not contain trivial loops, there is no loop incident to V_3 or V_4. Therefore all the labels of V_3 (and of V_4) are incident to edges that join V_1 or V_2. Let s be the number of vertices of G_S. Therefore, V_1 and V_2 are incident to more than $4S$ edges (since there is also the edges of σ), which is impossible. □

Let B be the bigon that contains V_3 and V_4. Let $B^* = \hat{S} - \operatorname{int} B$. Then B^* contains the edges of σ and V_1, V_2. Let J be the 3-ball of V_r, bounded by V_1 and V_2, which does not contain V_3 (and V_4).

We consider now the regular neighbourhood W of $B^* \cup J$. Then W is a solid torus, pierced twice by K_r. Let D be the disk face of G_S bounded by σ. Thus, the regular neighbourhood $N(W \cup D)$ is a punctured lens space. So its boundary $R = \partial N(W \cup D)$ is an essential 2-sphere, otherwise $E(K)(r)$ should be a lens space, which is an irreducible 3-manifold. Consequently, $\hat{Q}$ is not a minimal essential 2-sphere, which is a contradiction.

Remark. The purpose of this remark is to underline that if the knot is cable then Proposition 1.3 (i) is not necessarily true. If K is a (n, m)-cable knot then $q = 2$, and there exists an incompressible Seifert surface P of Euler characteristic

$$\chi(P) = m(2(1 - g_c) - 1) + n - nm$$

where g_c is the genus of the companion, (for more details see [4]). Then the genus of P is $g = (1 - \chi(P))/2$, so

$$2g - 1 = -\chi(P) = nm - n + m(2g_c - 1)$$

and the reducing slope is nm (see [11]).

Proof of ii)

If p is a projective slope, and not a reducing slope, that means that $E(K)(p) = \mathbf{R}P^3$. Then K is not a cable knot, by [11]. Therefore, $|p| \leq 2g - 1$ by ii) of Theorem 1.1.

4. Comments and questions

After fixing a reducing slope r, q is the minimal geometric intersection number between essential 2-spheres in $M(r)$ and the core of the attached solid torus. We note that for the exterior of knots $q \neq 4$ holds, but this is not the case in general (see the example in [12]). Note also that the examples in [6, 12, 20] are hyperbolic manifolds.

Due to Gordon-Litherland [13], M is a called *a cabled manifold* if M contains a submanifold homeomorphic to *a cable space* $C(m,n)$ whose one boundary component is just ∂M. We can regard $C(m,n)$ as the exterior of a (m,n)-loop lying in a solid torus.

We are interested in knowing whether $q=2$ is a characterization of cabled manifolds, as it is the case for exteriors of knots.

Here are two examples of existence of essential annuli (one non-separating case and one separating) with M non-cabled.

First, consider the 3-torus $N = S^1 \times S^1 \times S^1$ and let K be an essential loop on a torus $S^1 \times S^1 \times \{z\}$. Then the exterior M of K in N contains an essential non-separating annulus, but M is not cabled.

Consider now the case where N is the union of two knot complements along their boundaries and K be a knot that lies in the common 2-torus. Then the exterior M of K contains an essential separating annulus, but M is not cabled.

So, the fact that $q=2$ does not imply that M is cabled, but what about the inverse?

QUESTION 4.1. *Assume that M is irreducible and that M is not $S^1 \times D^2$. Is the fact that M is cabled implies that $q=2$?*

If M is reducible, then clearly $q=0$. Moreover, if $M=E(K)$ where K is a $(2,1)$-cable knot of a trivial knot (running twice in longitudinal direction) then $M = S^1 \times D^2$ and is a cabled manifold. Furthermore ∂M is compressible, hence $q=1$.

Note that there exist irreducible cabled manifolds (M,T) which do not admit reducing slope. Consider a non-trivial hyperbolic knot exterior $E(K)$ and a cable space $C(m,n)$ (the exterior of a (m,n)-loop L lying in a solid torus V). Let $T=\partial N(L)$ and $T'=\partial V$ be the boundary components of $C(m,n)$. Let M be the union of $E(K)$ and $C(m,n)$, where $\partial E(K)$ is glued to T' so that meridian of $E(K)$ goes to the (m,n)-loop on T'. Therefore M is cabled, irreducible and $\partial M = T$.

Let r be the cabling slope on T (i.e. the slope defined by the cabling annulus in $C(m,n)$). Then r is the only candidate of reducing slopes for M, if we choose K as a suitable hyperbolic knot (by [11]). But $M(r) = L(m,n) \# E(K)(1/0) = L(m,n)$ which is irreducible. Therefore r is not a reducing slope, and so ∂M does not contain reducing slopes.

By Claim 3.1, we have seen that q can never be 4, for exteriors of knots. This result uses the fact that S^3 does not contain non-trivial torsions. Is it the same for homology spheres?

Conjecture 4.2. *Assume that M is the exterior of a knot in a homology 3-sphere. Assume that there exists a reducing slope r. Then the minimal intersection number between the core of the r-Dehn filling on M and an essential 2-sphere in $M(r)$, is not equal to four.*

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