# 第 16 回広島仙台整数論集会アブストラクト

**7 月 11 日 (火)**

10:35 – 11:25 三原 朋樹/Tomoki Mihara (東京工業大学/Tokyo Institute of Technology)

完備付値体値連続関数環の無限族の境界上の極小素イデアルであるような極大イデアルの存在について

On the existence of a maximal ideal which is also a minimal prime ideal in the growth of an infinite family of rings of continuous functions with values in complete valuation fields

We study the existence of a maximal ideal which is also a minimal prime ideal in Banach rings of the form $(\prod_{i\in I} \mathrm{C}_{\mathrm{bd}}(X_i, k_i))/(\widehat{\bigoplus}_{i\in I}\mathrm{C}_0(X_i, k_i))$ for families $(X_i)_{i\in I}$ and $(k_i)_{i\in I}$ of topological spaces and Banach fields. We note that the class of such Banach rings contains many interesting Banach rings such as $\mathrm{C}_{\mathrm{bd}}(X, k)/\mathrm{C}_0(X, k)$ for a topological space $X$ and complete valuation field $k$ and $(\prod_{i\in I} k_i)/(\widehat{\bigoplus}_{i\in I} k_i)$ for a family $(k_i)_{i\in I}$ of Banach fields. Mathematical objects like the direct product modulo the direct sum appear in various fields. Their properties of mysterious, and many of them are known to be independent of ZFC. In our talk, we determine the condition when Banach rings in the class admits a maximal ideal which is also a minimal prime ideal, and show examples of Banach rings in the class such that the existence of such a maximal ideal is independent of ZFC.

11:40 – 12:30 加藤 正輝/Masaki Kato (神戸大学/Kobe University)

二重余接関数の加法型公式

An addition type formula for the double cotangent function

The double cotangent function is defined by the logarithmic derivative of the double sine function and can be regarded as a generalization of the usual cotangent function. In this talk, we show that the double cotangent function satisfies an addition type formula. This formula includes the reciprocity laws of (classical and higher) Dedekind sums, Lerch's functional equation and Ramanujan's formula. Furthermore, by letting the period tend to infinity in the formula, we obtain Eie and Liaw's formula for the double Hurwitz zeta values.

13:55 – 14:45 峰 正博/Masahiro Mine(東京工業大学/Tokyo Institute of Technology)

Dedekind ゼータ関数の対数微分の値分布について

On the value-distributions of logarithmic derivatives of Dedekind zeta functions

The study of the value-distributions of zeta or $L$-functions is a classical topic in analytic number theory. We consider the distributions of values of logarithmic derivatives of Dedekind zeta functions on a fixed vertical line. The density functions of such value-distributions were studied by Kohji Matsumoto. However, it seemed difficult to give the density functions in the case of non-Galois number fields. In this talk, we determine the density functions of the value-distributions for any algebraic number field by the method different from Matsumoto. Applying almost the same proof, we also prove a similar result for $L$-functions associated with primitive cusp forms for a congruence subgroup.

15:00 – 15:50 山田 智宏/Tomohiro Yamada (大阪大学/Osaka University)

Recent results on odd perfect numbers and related numbers

I would like to speak about some results on odd perfect numbers and related numbers. For example, if $\sigma^{**}(n)$ denotes the sum of bi-unitary divisors of an integer $n$, then $n = 2$

and 9 are the only integers such that $\sigma^{**}(\sigma^{**}(n)) = 2n$.

16:05 – 16:55 村原 英樹/Hideki Murahara (中村学園大学/Nakamura Gakuen University)

On algebraic relations among finite multiple zeta values

In recent years, two types of finite versions of multiple zeta values, $\mathcal{A}$-finite multiple zeta values and symmetrized multiple zeta values have been investigated. The $\mathcal{A}$-finite multiple zeta value is a collection of certain finite sums whose setting was given by D. Zagier. The symmetrized multiple zeta value was introduced by M. Kaneko and Zagier to establish a crucial bridge between the multiple zeta values and $\mathcal{A}$-finite multiple zeta values.

For these two types of finite multiple zeta values, Kaneko and Zagier conjectured that the algebra generated by $\mathcal{A}$-finite multiple zeta values and the algebra generated by symmetrized multiple zeta values are isomorphic. From this point of view, we conjectured exactly the same equations hold for $\mathcal{A}$-finite multiple zeta values and symmetrized multiple zeta values. In this talk, we mainly focus on the algebraic relations among these two types of finite multiple zeta values.

17:10 – 18:00 佐藤 信夫/Nobuo Sato (京都大学/Kyoto University)

On a new class of linear relations among the multiple zeta values

In this talk, we consider iterated integrals on a projective line minus generic four points and introduce a new class of linear relations among the MZVs, which we call *confluent relations*. We start with Goncharov's notation for iterated integrals, review some basic notions and properties of iterated integrals, and define a class of relations among iterated integrals, which naturally arise as "solving differential equations step by step". Confluent relations are defined as the limit of these relations when merging two out of the four punctured points. One of the significance of the confluent relations is that it is a rich family and seems to exhaust all the linear relations among the MZVs. As a good reason for this, we show that confluent relations imply the extended double shuffle relations as well as the duality relations. This is a joint work with Minoru Hirose at Kyushu University.

**7 月 12 日 (水)**

9:30 – 10:20 Yu Yang (京都大学/Kyoto University)

$p$-average theorem and anabelian geometry of curves over algebraically closed fields of characteristic $p > 0$

Let $X$ be a pointed stable curve over an algebraically closed field of characteristic $p > 0$. In this talk, I will prove that the semi-graph of anabelioids of PSC-type associated to $X$ can be reconstructed group-theoretically from the geometric log fundamental group of $X$. This result may be regarded as a mono-anabelian version of the combinatorial Grothendieck conjecture in positive characteristic. As an application, we obtain that, if the geometric genus of each irreducible component of $X$ is 0, then the isomorphism class of the geometric log fundamental group of $X$ completely determines the isomorphism class of $X$ as a scheme. This result generalizes a result of A. Tamagawa to the case of (possibly singular) pointed stable curves.

10:35 – 11:25 更科 明/Akira Sarashina (京都大学/Kyoto university)

正標数代数曲線の同型類の幾何的基本群による復元

Reconstruction of curves in positive characteristic by their geometric fundamental groups

Our main question is whether the isomorphism class as a scheme of a curve $X$ over $\overline{\mathbb{F}}_p$ can be reconstructed by the étale fundamental group of $X$. Tamagawa answered this

question affirmatively when the genus of $X$ is 0. In this talk, we will discuss the genus 1 case, and prove a similar result when the genus of $X$ is 1, the cardinality of $X^{cpt}\backslash X$ is 1, and $p \neq 2$.

11:40 – 12:30 辻村 昇太/Shouta Tsujimura (京都大学/Kyoto University)

Geometric version of the Grothendieck conjecture for universal curves over Hurwitz stacks

Hurwitz stacks are algebraic stacks that parametrize simple coverings. In this talk, we consider a certain geometric version of the Grothendieck Conjecture for universal curves over Hurwitz stacks. This result generalizes a similar result obtained by Hoshi and Mochizuki in the case of universal curves over moduli stacks of pointed smooth curves.

In the first part of this talk, we introduce the notion of "profiled log Hurwitz stacks" (i.e., log algebraic stacks that parametrize Hurwitz coverings for which the marked points are equipped with a certain ordering determined by combinatorial data which we refer to as a "profile") and examine fundamental geometric properties of these stacks. In the second part of this talk, we sketch the proof of the above version of the Grothendieck Conjecture in the hyperelliptic case. This proof proceeds by applying techniques from combinatorial anabelian geometry to various objects that arise from profiled log Hurwitz stacks.

13:55 – 14:45 村上 和明/Kazuaki Murakami (慶應義塾大学/Keio University)

On an upper bound of $\lambda$-invariants of $\mathbb{Z}_p$-extensions over an imaginary quadratic field

Let $p$ be an odd prime number. In this talk, we give an explicit upper bound of $\lambda$-invariants for all $\mathbb{Z}_p$-extensions of an imaginary quadratic field $k$ under several assumptions. Especially, we give an explicit upper bound of $\lambda$-invariants for all $\mathbb{Z}_p$-extensions of $k$ in the case where the $\lambda$-invariant of the cyclotomic $\mathbb{Z}_p$-extension of $k$ is equal to 3.

15:00 – 15:50 広瀬 稔/Minoru Hirose (九州大学/Kyushu University)

On Gross's refined class number formula and enhanced Stickelberger elements

In 1988, Gross proposed a conjectural congruence between Stickelberger elements and algebraic regulators, which is often referred to as the refined class number formula. The speaker proved this conjecture last year. The method is based on the construction of a certain lift of Stickelberger element called an enhanced Stickelberger element. In this talk, we explain the method of the proof and some related topics.

16:05 – 16:55 熱田 真大/Mahiro Atsuta (慶應義塾大学/Keio University)

Iwasawa theory for $p = 2$

Let $p$ be a prime, $F$ a CM field, and $F_\infty$ the cyclotomic $\mathbf{Z}_p$-extension of $F$. If $p$ is an odd prime, it is well known that the minus part of the unramified Iwasawa module $X_{F_\infty}^-$ has no non-trivial finite $\mathbf{Z}_p[[\mathrm{Gal}(F_\infty/F)]]$-submodule. But for $p = 2$, Ferrero proved that $F$ is an imaginary quadratic field, $X_{F_\infty}$ has non-trivial finite $\mathbf{Z}_2[\mathrm{Gal}(F_\infty/F)]]$-submodule. In this talk, we discuss classical Iwasawa theory for $p = 2$ and generalize Ferrero's result for a CM field.

17:10 – 18:00 丹下 稜斗/Ryoto Tange (九州大学/Kyushu University)

On adjoint $\mathcal{L}$-invariants for holonomy representations of hyperbolic knots

We introduce the adjoint $\mathcal{L}$-invariant for the holonomy representation of a hyperbolic knot, which may be seen as a topological analogue of Greenberg's $\mathcal{L}$-invariant for the adjoint Galois representation. We then explain some relations of our $\mathcal{L}$- invariants with Porti's torsion functions and cusp shapes. These relations may be regarded as topological analogues of the relations of Greenberg's $\mathcal{L}$-invariant with adjoint Selmer modules and

Tate periods.

**7 月 13 日 (木)**

9:30 – 10:20 奥村 喜晶/Yoshiaki Okumura (東京工業大学/Tokyo Institute of Technology)

Rasmussen-Tamagawa 予想の関数体類似について
A function field analogue of the Rasmussen-Tamagawa conjecture

Let $K$ be a global function field over $\mathbb{F}_q$ and $\mathbb{G}_a$ the additive group over $K$. A Drinfeld module over $K$ is an $\mathbb{F}_q$-algebra homomorphism $\varphi : \mathbb{F}_q[t] \to \mathrm{End}_{\mathbb{F}_q}(\mathbb{G}_a)$ with some conditions, which is defined by Drinfeld in 1974 as an analogue of elliptic curves over number fields. In this talk, for Drinfeld modules over $K$, we will discuss a function field analogue of a non-existence conjecture for abelian varieties over number fields suggested by Rasmussen and Tamagawa, and introduce some results about it.

10:35 – 11:25 宮崎 弘安/Hiroyasu Miyazaki (理化学研究所/RIKEN)

冪零 $K$-群のサイクル理論的な対応物と加群構造
A cycle-theoretic counterpart of the nilpotent $K$-group and its module structure

The nilpotent $K$-group is defined as an obstruction to $\mathbb{A}^1$-homotopy invariance of Quillen's higher $K$-group. C. Weibel proved that the nilpotent $K$-group has a module structure over the big Witt ring $\mathbb{W}(k)$ of the base field $k$. Using the module structure, he obtained several vanishing theorems on the nilpotent $K$-group. In this talk, as a cycle-theoretic counterpart of the nilpotent $K$-group, we introduce the "nilpotent higher Chow group with modulus." It is an obstruction to $\mathbb{A}^1$-homotopy invariance of Binda-Saito's higher Chow group with modulus. We also construct its $\mathbb{W}(k)$-module structure, and prove vanishing theorems analogous to Weibel's results.

11:40 – 12:30 望月 哲史/Satoshi Mochizuki (中央大学/Chuo University)

A motivic cohomology theory for exact categories

In the lecture, I will propose a definition of bigraded cohomology theory $\mathrm{H}^{p,q}(-)$ on the category of small exact categories which is a localizing invariant with respect to the first degree and I call it a motivic cohomology theory for exact categories.

I will justify the naming by showing the following facts.

1. I will construct an Atiyah-Hirzebruch type spectral sequence whose $E_2$-terms are motivc cohomology theory of an exact category $\mathcal{E}$ and it converges to algebraic $K$-theory of $\mathcal{E}$.
2. If $\mathcal{E}$ is a category of vector bundles on a smooth scheme $X$ over a perfect field, then there exists a canonical isomorphism between $\mathrm{H}^{p,q}(\mathcal{E})$ and Voevodsky's motivic cohomology $\mathrm{H}^{p,q}(X) = \mathrm{H}^p_{\mathcal{M}}(X, \mathbb{Z}(q))$.

13:55 – 14:45 境 優一/Yuichi Sakai (九州大学/Kyushu University)

Modular forms of certain rational weights and modular linear differential equations

In recent years, we find some relations between modular forms and characters of specific vertex operator algebras(VOAs) by using modular linear differential equations(MLDEs). As a well-known example, the Kaneko-Zagier equation which is a MLDE of second-order appears in connection with both supersingular $j$-invariants of elliptic curves and a classification of (2D) conformal field theory. In this talk, by focusing on the simple Virasoro VOAs(minimal models), we give a certain correspondence between characters for such

VOAs and modular forms of certain rational weights. Also, we give a result with respect to the order of MLDEs for modular forms of rational weights. This is a joint work with K. Nagatomo(Osaka university).

15:00 – 15:50 町出 智也/Tomoya Machide (国立情報学研究所/JST)

生成関数による４重ゼータ値の和公式

Sum formula for quadruple zeta values by use of the generating function

Multiple zeta values (MZVs) have been actively studied since more than two decades. The sum formula, an outstanding example of their linear relations, says that the sum of all MVZs of fixed depth and weight is equal to the special value of the Riemann zeta function. In this talk, we generalize the sum formula, in the case of quadruple zeta values (MZVs of depth 4), by use of the generating function with the action of the matrix ring. Substituting appropriate values for the variables, we obtain its weighted analogues, which include known and new results.

16:05 – 16:55 生駒 英晃/Hideaki Ikoma (京都大学/Kyoto University)

Adelic Cartier divisors with base conditions and the Bonnesen-Diskant-type inequalities

The arithmetic volume of an adelic Cartier divisor $\overline{D}$ counts the asymptotic number of the small sections with respect to the high tensor powers of $\overline{D}$. The existence problem of small sections is of fundamental importance in Arakelov geometry, and is related to the construction problem of auxiliary functions in transcendental number theory.

In this talk, I introduce a notion of pairs of adelic $\mathbb{R}$-Cartier divisors and $\mathbb{R}$-base conditions, and study fundamental properties of the arithmetic volume function defined for such pairs. I explain that the Gateaux derivatives of the arithmetic volume function at big pairs along the directions of adelic $\mathbb{R}$-Cartier divisors are given by suitable arithmetic positive intersection numbers. As a corollary, I obtain an Arakelov theoretic analogue of the classical Bonnesen-Diskant inequality in convex geometry. If time permits, I would like to explain with examples how our notion of pairs is related to the study of the positivity of the usual adelic $\mathbb{R}$-Cartier divisors, and future perspectives.

17:10 – 18:00 松澤 陽介/Yosuke Matsuzawa (東京大学/Tokyo University)

On the arithmetic degrees of endomorphisms on surfaces

For a dominant rational self-map of an algebraic variety, one can define an invariant, called (first) dynamical degree, which measures the asymptotic behavior of the iterates of the map. On the other hand, when the variety is defined over a number field, one can associate to an orbit an invariant using Weil height function, called arithmetic degree, which measures the arithmetic complexity of the orbit. Kawaguchi and Silverman conjectured that the first dynamical degree and the arithmetic degree of a Zariski dense orbit coincide. I explain the proof of this conjecture for endomorphisms on surfaces and investigate arithmetic degrees of these maps. This is partially joint work with Kaoru Sano and Tkahiro Shibata.

**7 月 14 日 (金)**

9:30 – 10:20 堀永 周司/Shuji Horinaga (京都大学/Kyoto University)

概正則保型形式の生成する表現について

On the representations generated by nearly holomorphic modular forms

Shimura defined the notion of nearly holomorphic modular forms and proved various properties of such forms. In recent years, Pitale-Saha-Schmidt found a representation

theoretic aspect of nearly holomorphic automorphic forms. They calculated the representations, which occur in the space of nearly holomorphic modular forms on the groups, and their multiplicities. The key points of their theory are the theory of parabolic BGG category $\mathcal{O}^{\mathfrak{p}}$ and the vanishing theorem of nearly holomorphic automorphic forms. In this talk, I will discuss the generalization of their theorems and obstructions.

10:35 – 11:25 鈴木 美裕/Miyu Suzuki (京都大学/Kyoto University)

On generalized Shalika models for representations of $\mathrm{SO}(4n)$

Jiang and Qin introduced the notion of a generalized Shalika model for representations of $\mathrm{SO}(4n)$. They found that this model has a relation to existence of a pole of Eisenstein series. In this talk, we will consider the case of unramified principal series representations of $p$-adic $\mathrm{SO}(4n)$ and determine the condition for existence of this model. We also consider relations between generalized Shalika models and certain linear models under theta lifts.

11:40 – 12:30 武田 渉/Wataru Takeda (京都大学/Kyoto University)

Relatively $r$-prime lattice points in all abelian extensions

Let $K$ be a number field and let $\mathcal{O}_K$ be its ring of integers. We regard an $m$-tuple of ideals $(\mathfrak{a}_1, \mathfrak{a}_2, \ldots, \mathfrak{a}_m)$ of $\mathcal{O}_K$ as a lattice point in $K^m$. We say that a lattice point $(\mathfrak{a}_1, \mathfrak{a}_2, \ldots, \mathfrak{a}_m)$ is relatively $r$-prime, if there exists no prime ideal $\mathfrak{p}$ such that $\mathfrak{a}_1, \mathfrak{a}_2, \ldots, \mathfrak{a}_m \subset \mathfrak{p}^r$.

We estimate the number of relatively $r$-prime lattice points in $K^m$ with their components having norm less than $x$. We use the bounds of Dedekind zeta functions on the critical line. We show some conditional results for all number fields and unconditional results for abelian extensions with degree less than or equal to 6. This is partially joint work with Shinya Koyama.

13:55 – 14:45 佐野 薫/Kaoru Sano (京都大学/Kyoto University)

The canonical height functions for Jordan blocks and some application

In the field of Diophantine geometry, the height functions are important functions which measure the arithmetic complexity of rational points. Kawaguchi and Silverman made the canonical height functions for Jordan blocks which are useful tools to understand the speed of growth of the ample height of rational point under iterating an endomorphism. I refined the canonical height functions for Jordan blocks and found the explicit order of the speed of growth of the ample height. In my talk, I explain the canonical height function and some interesting application.

15:00 – 16:00 服部 新/Shin Hattori (九州大学/Kyushu University)

Coleman-Mazur 固有値曲線の次数有限な既約成分は重さ空間上有限
Irreducible components of the eigencurve of finite degree are finite over the weight space

Let $p$ be a rational prime and $N$ a positive integer prime to $p$. The Coleman-Mazur eigencurve $\mathcal{C}_N$ is a rigid analytic curve over $\mathbb{Q}_p$ which $p$-adically interpolates the set of classical elliptic eigenforms of finite slope and tame level $N$. In this talk, I will explain a joint work with James Newton in which we show that any irreducible component of $\mathcal{C}_N$ of finite degree over the weight space $\mathcal{W}$ is actually finite over $\mathcal{W}$. By a theorem of Chenevier, our theorem plus a conjecture on slopes near the boundary of $\mathcal{W}$ (due to Coleman-Mazur, Buzzard-Kilford) implies that any such component should be in the ordinary locus. If time permits, I will give a brief introduction to recent topics on slopes of modular forms.