Galois representations in fundamental groups and their Lie algebras

Makoto MATSUMOTO

We treat the following subjects (possibly not all). A fundamental reference to the fundamental group is SGA1 ("Revêtement Etales et Groupe Fondamental" by A. Grothendieck and Mme M. Raynaud, 1971).

I Algebraic fundamental group.

Definition of algebraic fundamental group. Use of "etale coverings" and "fiber functors", instead of the usual notion of "path modulo homotopy."

Two keys:

- Comparison theorem with the topological fundamental groups. This allows us to study algebraic fundamental groups using classical topology.
- The algebraic fundamental group of a field is its absolute Galois group. This connects algebraic fundamental groups with number theory.

II Galois representation on fundamental groups, as monodromy.

The absolute Galois group of the base field acts on the algebraic fundamental group. This can be regarded as an analogy to the geometric monodromy.

III Computation using Puiseux Series.

Compute the Galois action using analytic continuation. Introduce the notion of tangential base points (and tangential morphisms).

IV Soulé's cocycle and Deligne-Ihara conjecture.

Formulate Deligne-Ihara's conjecture about the Galois-action on the Lie algebra of the fundamental group of projective line minus three points. Explain briefly about a partial solution to this conjecture.

A possible subject to work is to give a computational direct proof of the fact that Soulé's cocycle appears in the representation on the fundamental group of projective line minus three points.