Four Dimensional Topology

November 15 – November 17, 2010
Hiroshima University

Program and Abstract

November 15 (Mon)

14:00–14:40

On the unknotting conjecture in dimension four VI

Takao Matumoto (Hiroshima University)

The conjecture was reduced to the special case of 1-parameter family with birth and death of one double point which can be expressed by a 1-papremeter family of simple surface braids. We explain the final step of proof which uses an induction argument by analyzing the trace of double point on the chart diagram of trivial 2-dimensional sphere knot.

14:50–15:30

On the Jones polynomial of ribbon knots

Toshifumi Tanaka (Gifu University)

A symmetric union introduced by Kinoshita and Terasaka in 1957 is an example of a slice knot which bounds a disc in the four-ball. It is easy to see that every symmetric union is a ribbon knot, but the converse is still an open question. In this talk, we give a formula of the Jones polynomial of a symmetric union to investigate this question. We also give special properties of the first derivative of the Jones polynomial of a symmetric union.

16:00–16:40

Broken Lefschetz fibrations and pencils

Refik İnanç Baykur (Brandeis University)

This will be an introductory talk on broken Lefschetz fibrations and pencils on smooth four-manifolds.

16:50–17:30

On diffeomorphisms over non-orientable surfaces standardly embedded in the 4-sphere and the level-2 mapping class group

Susumu Hirose (Saga University)

For the orientable closed surface standardly embedded in the 4-sphere, it was shown that a diffeomorphism over this surface is extendable to the 4-sphere if and only if this diffeomorphism preserves the Rokhlin quadratic form of this surface. In this talk, I will explain a plan of investigation on the same phenomenon for the non-orientable closed surface stadardly embedded in the 4-sphere.

17:40–18:20

Seiberg-Witten theory and intersection forms with local coefficients

Nobuhiro Nakamura (University of Tokyo)

We introduce a variant of the Seiberg-Witten equations, $\mathrm{Pin}^-(2)$-monopole equations, and explain its applications to intersection forms with local coefficients of 4-manifolds. The first application is an analogue of Froyshov's results on 4-manifolds which have definite forms with local coefficients. The second one is a local coefficient version of Furuta's 10/8-inequality. As a corollary, we construct nonsmoothable spin 4-manifolds satisfying Rohlin's theorem and the 10/8-inequality.

November 16 (Tue)

09:00–09:50

Periodic-end Dirac operators and Seiberg-Witten theory

Daniel Ruberman (Brandeis University)

We study an extension of Seiberg-Witten invariants to 4-manifolds with the homology of $S^1 \times S^3$. This extension has many potential applications in low-dimensional topology, including the study of the homology cobordism group. Because $b_2^+ = 0$, the usual strategy for defining invariants does not work–one cannot disregard reducible solutions, and the count of solutions can jump in a family of metrics. To remedy this, we define an index-theoretic counter-term that jumps by the same amount. The counterterm is the index of the Dirac operator on a manifold with a periodic end modeled at infinity by the infinite cyclic cover of the manifold. This is joint work with Tomasz Mrowka and Nikolai Saveliev.

10:00–10:40

Round handles, logarithmic transforms, and smooth four-manifolds

Refik İnanç Baykur (Brandeis University)

In this talk, we will unfold the strong affiliation of round handles with smooth four-manifolds. Several essential topics that appear in the study of smooth four-manifolds, such as logarithmic transforms along tori, exotic smooth structures, cobordisms, handlebodies, broken Lefschetz fibrations, one and all, will come into play as we discuss the relevant interactions between them.

11:00–11:40

Elimination of definite fold and broken Lefschetz fibrations

Osamu Saeki (Kyushu University)

In this survey talk, I will first present my own result concerning the elimination of definite folds for generic maps of 4-manifolds into surfaces. Using this, I show that every closed oriented 4-manifold admits a broken Lefschetz fibration (BLF) over the 2-sphere, which is a result due to Baykur. Given a 4-manifold, there are a lot of such structures, and I will present Williams' recent result on a set of certain moves for BLFs that is enough to describe the difference of any two homotopic BLFs. The essential part in the proof lies in the elimination of definite folds for 1-parameter families of generic maps.

11:50–12:30

Lefschetz fibrations over a torus admitting a section of square 0

Naoyuki Monden (Osaka University)

We show that there exists a family of infinitely many Lefschetz fibrations over a torus admitting a section of square 0 which cannot be decomposed as fiber sums. Moreover, there is no upper bounds on the number of singular fibers of such a Lefschetz fibration.

14:00–14:40

Cork twisting exotic Stein 4-manifolds

Kouichi Yasui (Kyoto University)

From any given 4-dimentional oriented handlebody without 3- and 4-handles and with $b_2 > 0$, we construct arbitraly many compact Stein 4-manifolds so that they are mutually homeomorphic but not diffeomorphic and that the fundamental groups, the homology groups, the homology groups of the boundary, and the intersection forms of them are isomorphic to those of the given handlebody. If the time permits, we also discuss exotic embeddings of 4-manifolds. This is a joint work with Selman Akbulut.

14:50–15:30

4-fold symmetric quandle invariants of 3-manifolds
(joint work with Eri Hatakenaka)

Takefumi Nosaka (RIMS, Kyoto University)

We introduce 4-fold symmetric quandles, and a 4-fold symmetric quandle homotopy invariant of closed 3-manifolds. This invariant is valued in a group ring of a quotient of the homotopy group of the quandle space. We classify 4-fold symmetric quandles, and show that a category composed of them is equivalent to the category of pointed groups. Using oriented bordism groups, our invariant turns, out to be at least as strong as Dijkgraaf-Witten invariants. We also give an application of a cocycle invariant.

16:00–16:40

Quandle cocycle invariant of a certain T^2-link

Inasa Nakamura (RIMS, Kyoto University)

We consider a surface link which is presented by a simple branched covering over the standard torus, which we call a torus-covering link. We present the quandle cocycle invariant of a certain torus-covering T^2-link, by using the quandle cocycle invariants of the closure of a classical m-braid.

16:50–17:30

Hurwitz equivalence in braid systems and its applications to surface braids

Yoshiro Yaguchi (Hiroshima University)

Hurwitz action of the n braid group B_n on the n-fold direct product of a group G is studied. Hurwitz action can be used in study of braided surfaces, surface braids and orientable surface links. In this talk, we will find infinite orbits in G^4 when G is the semi-direct product $S_m \ltimes Z^m$, where S_m is the symmetric group of degree m and Z^m is the m-fold direct product of the cyclic group Z. We will also give its applications to surface braids with 4 branch points.

17:40–18:20

Recognition criteria for singularities of fronts and their applications

Kentaro Saji (Gifu university)

In this talk, we deal with criteria for determining $D_4^{\pm}$ singularities of wave fronts.

The unit cotangent bundle $T_1^*\boldsymbol{R}^{n+1}$ of $\boldsymbol{R}^{n+1}$ has the canonical contact structure and can be identified with the unit tangent bundle $T_1\boldsymbol{R}^{n+1}$. Let α denote the canonical contact form on it. A map $i : M \to T_1\boldsymbol{R}^{n+1}$ is said to be *isotropic* if $\dim M = n$ and the pull-back $i^*\alpha$ vanishes identically. An isotropic immersion is called a *Legendrian immersion*. We call the image of $\pi \circ i$ the *wave front set* of i, where $\pi : T_1\boldsymbol{R}^{n+1} \to \boldsymbol{R}^{n+1}$ is the canonical projection. We denote by $W(i)$ the wave front set of i. Moreover, i is called the *Legendrian lift* of $W(i)$. With this framework, we define the notion of fronts as follows: A map-germ $f : (\boldsymbol{R}^n, \mathbf{0}) \to (\boldsymbol{R}^{n+1}, \mathbf{0})$ is called a *wave front* or a *front* if there exists a unit vector field ν of $\boldsymbol{R}^{n+1}$ along f such that $L = (f, \nu) : (\boldsymbol{R}^n, \mathbf{0}) \to (T_1\boldsymbol{R}^{n+1}, \mathbf{0})$ is a Legendrian immersion.

The main result is as follows:

Theorem 1 ([1]) *Let $f(u,v) : (\boldsymbol{R}^2, \mathbf{0}) \to (\boldsymbol{R}^3, \mathbf{0})$ be a front and (f, ν) its Legendrian lift. The germ f at $\mathbf{0}$ is a D_4^+ singularity (resp. D_4^- singularity) if and only if the following two conditions hold.*

(a) The rank of the differential map $df_{\mathbf{0}}$ is equal to zero.

(b) $\det(\operatorname{Hess}\lambda(\mathbf{0})) < 0$ *(resp.* $\det(\operatorname{Hess}\lambda(\mathbf{0})) > 0$*),*

where $\lambda(u,v) = \det(f_u, f_v, \nu)$, $f_u = df(\partial/\partial u)$, $f_v = df(\partial/\partial v)$ and $\det(\operatorname{Hess}\lambda(\mathbf{0}))$ means the determinant of the Hessian matrix of λ at $\mathbf{0}$.

A map-germ right-left equivalent to $(u, v) \mapsto (uv, u^2 + 3\varepsilon v^2, u^2 v + \varepsilon v^3)$ at $\mathbf{0}$ is called a D_4^+ *singularity* if $\varepsilon = 1$ (resp. a D_4^- *singularity* if $\varepsilon = -1$). Since our criteria require only the Taylor coefficients of the given germ, this can be useful for identifying the $D_4^\pm$ singularities on explicitly parameterized maps.

To prove of this theorem, we regard $D_4^\pm$ singularities as 3-dimensional slices of 4-dimensional $D_4^\pm$ singularities. A map-germ $(\boldsymbol{R}^3, \mathbf{0}) \to (\boldsymbol{R}^4, \mathbf{0})$ right-left equivalent to $(u, v, t) \mapsto (uv, u^2 + 2tv \pm 3v^2, 2u^2 v + tv^2 \pm 2v^3, t)$ at $\mathbf{0}$ is called a 4-*dimensional* $D_4^\pm$ *singularity*, respectively.

As an application, we investigate behaviors of singular curvatures of cuspidal edges near D_4^+ singularities.

References

[1] K. Saji, *Criteria for D_4 singularities of wave fronts*, preprint.

November 17 (Wed)

09:00–09:40

Instanton Floer homology for lens spaces

Hirofumi Sasahira (Nagoya university)

In this talk, I will introduce instanton Floer homology for lens spaces. I give examples of calculation for some lens spaces.

09:50–10:30

Knot surgery on $S^2 \times S^2$

Motoo Tange (RIMS, Kyoto University)

$S^2 \times S^2$ contains the cusp neighborhood. Any knot surgery on $S^2 \times S^2$ is standard. As some applications of the argument we show that Scharlemann's manifolds are standard.

10:50–11:30

On Cappell-Shaneson homotopy spheres

Shohei Yamada (Osaka University)

Cappell and Shaneson constructed a family of (smooth) homotopy 4-spheres by surgering mapping tori of 3-torus. It has been studied for about 30 years whether the homotopy spheres are diffeomorphic to S^4. The problem is, however, unsolved in most cases. Gompf developed a new method for it, which does not require Kirby calculus. In this talk, we give a brief introduction to the method and the result by Gompf. After that, we show that some more Cappell-Shaneson homotopy spheres are diffeomorphic to S^4. This improves Gompf's result. We also give an application of algebraic number theory to the problem with computer calculation (by PARI/GP).

11:40–12:20

Immersed link cobordism and multi-variable Alexander polynomial

Akio Kawauchi (Osaka City University)

When a link L is cobordant to a link L' by a cobordism of immersed annuli, we relate the number of the double points of the immersed annuli to the beta-ranks $\beta(L), \beta(L')$ and the torsion multi-variable Alexander polynomials $A^T(L), A^T(L')$ of L and L'.